CHAPTER - 1
REAL NUMBERS

Fundamental Theorem of Arithmetic :

Every composite number can be expressed (factorised) as a product of Primes, and this factorization is unique, apart from the order in which the prime factors occur.

Ex : 30 = 2 x 3 x 5

LCM and HCF : If a and b are two positive integers. Then the product of a, b is equal to the product of their LCM and HCF.

LCM x HCF = a x b

To find LCM and HCF of 12 and 18 by the prime factorization method.

$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

HCF of 12 and 18 = $2^1 \times 3^1 = 6$

(product of the smallest powers of each common prime factors in the numbers)

LCM of 12 and 18 = $2^2 \times 3^2 = 36$

(product of the greatest powers of each prime factors, in the numbers)

product of the numbers = 12 x 18 = 216

LCM x HCF = 36 x 6 = 216

$\therefore$ Product of the numbers = LCM x HCF

Natural numbers set N = {1, 2, 3, 4, ...}

Whole numbers set W = {0, 1, 2, 3, ...}

Intergers Z (or) I = {.........–3, –2, –1, 0, 1, 2, 3.........}

Rational Numbers (Q) : If p, q are whole numbers and $q \neq 0$ then the numbers in the form of $\frac{p}{q}$ are called Rational Numbers.

Rational Numbers Set $Q = \left\{ \frac{p}{q} / p, q \in z, q \neq 0 \ \ HCF(p, q) = 1 \right\}$

All rational numbers can be written either in the form of terminating decimals or non-terminating repeating decimals.

Ex : $\frac{2}{7}, \frac{5}{2}, -4, 3, 0, \ldots\ldots$

Between two distinct rational numbers there exist infinite number of rational numbers.

- A rational number between 'a' and 'b' $= \frac{a+b}{2}$

Terminating Decimals in Rational Numbers.

- Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$, where p and q are co-prime ; and the prime factorization of q is of the form $2^n.5^m$, where n, m are non-negative integers.

Conversely, Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n.5^m$, where n, m are non-negative integers. Then x has a decimal expansion which is terminal.

Ex : In the rational number $\frac{3}{40}$, p = 3, q = 40.

q = 40 = 2 x 2 x 5 = 2^2 x 5^1 in the form of $2^n.5^m$.

$\therefore$ $x = \frac{3}{40} = \frac{3}{2^2 \times 5^1} = \frac{15}{2^2 \times 5^2} = \frac{15}{(10)^2} = \frac{15}{100} = 0.15$ is in the form of terminating decimal and

Non - Terminating, Recurring decimals in Rational Numbers :

Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form $2^n.5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

Ex : In the rational number $\frac{11}{30}$, p = 11, q = 30.

q = 30 = 2 x 3 x 5, is not in the form of $2^n.5^m$.

is non-terminating, repeating decimal.

Irrational Numbers (Q') : The numbers ca… are called irrational numbers.
The decimal expansion of every irrational number is non-terminating and non-repeating.

Ex :

- An irrational number between a and b $= \sqrt{ab}$
- $\sqrt{p}$ is irrational, where p is prime.

Ex :

- Let p be a prime number. Let p divide a^2. Then p divides a, where a is a positive integer.
- Sum (or difference) of a rational number and irrational number is an irrational number.
- Product (or quotient) of a non-zero rational and an irrational number is an irrational number.
- The sum of the two irrational numbers need not be irrational.

Ex : are irrational but $\sqrt{2} + (-\sqrt{2}) = 0$ which is rational.

- Product of two irrational numbers need not be irrational.

Ex : are irrational but , which is rational.

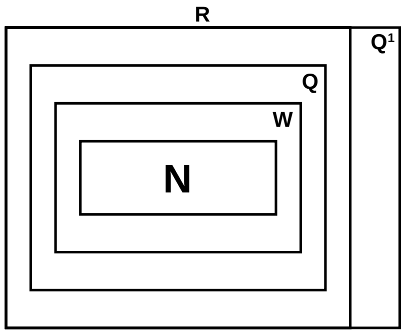
Real Numbers (R) : The set of rational and irrational numbers together are called real numbers.

- Between two distinct real numbers there exists infinite number of real numbers.
- Between two distinct real numbers there exist infinite number of rational and irrational number.
- With respect to addition real numbers are satisfies closure, commutative, associative, identity, inverse and distributive properties. Here '0' is the additive identity and additive inverse of a is – a.
- With respect to multiplication, non-zero real numbers are satisfies closure, commutative, associative, identity, inverse properties.

  Here '1' is the multiplicative identity.

  For $a (\neq 0) \in R$, $\frac{1}{a}$ is the multiplicative inverse of 'a'

- $N \subset W \subset Z \subset Q \subset R$



- **Logarithms :**
- Logarithms are used for all sorts of calculation in engineering, science, business and economics.
- If $a^n = x$, we write it as $\log_a x = n$, where a and x are positive numbers and $a \neq 1$.
- Logarithmic form of $a^n = x$ is $\log_a x = n$.
- Exponential form of $\log_4 64 = 3$ is $4^3 = 64$.
- The logarithms of the same number to different bases are different.

  Ex : $\log_4 64 = 3$, $\log_8 64 = 2$.
- The logarithm of 1 to any base is zero i.e., $\log_a 1 = 0$, $\log_2 1 = 0$.
- The logarithm of any number to the same is always one.

  i.e., $\log_a a = 1$, $\log_{10} 10 = 1$.
- **Laws of Logarithms :**

  1) $\log_a xy = \log_a x + \log_a y$

  2) $\log_a \frac{x}{y} = \log_a x - \log_a y$

  3) $\log_a x^m = m.\log_a x$
- The logarithm of a number consists of two parts.

  i) The integral part of the logarithm *(characteristic).*

  ii) The fractional or decimal part of the logarithm *(Mantissa).*

  Ex : $\log_{10} 16 = 1.2040$

  Characteristic = 1

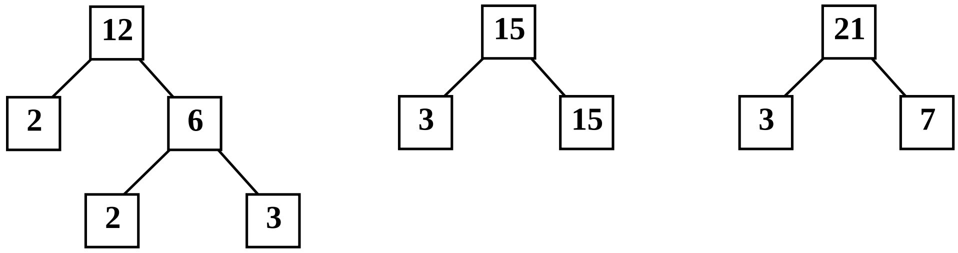
  Mantissa = 0.2040

TWO MARK QUESTIONS

1. Find the LCM and HCF of the following integers.

   i) 12, 15 and 21 *(problem solving)*

A. The given integers 12, 15 and 21.



$\therefore$ 12 = 2 x 2 x 3 $\qquad$ $\therefore$ 15 = 3 x 5 $\qquad$ $\therefore$ 21 = 3 x 7

HCF of 12, 15 and 21 = product of the smallest powers of each common prime factors in the numbers.

$= 3^1 = 3$

LCM of 12, 15 and 21 = product of the greatest powers of each prime factors in the numbers.

$= 2^2 \text{ x } 3^1 \text{ x } 5^1 \text{ x } 7^1 = 420$

2. Without actually performing long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion. *(communication)*

   i) $\frac{13}{3125}$, ii)

$$\frac{13}{25} = \frac{13}{5\times5} = \frac{13\times2^2}{5^2\times2^2} = \frac{13\times4}{10^2} = \frac{52}{10^2} = 0.52$$

A.

Here, $q = 5^5$, which is of the form 2^n5^m (n = 0)

Hence, the given rational number has a terminating decimal expansion.

ii)

Here $q = 2^6 \text{ x } 5^2$ which is of the form $2^n.5^m$ (n = 6, m = 2).

So, the given rational number has a terminating decimal expansion.

3. **Write the decimal expansion of the following rationals.** ***(communication)***

   i) , ii)

A. i)

ii) $\frac{143}{110} = \frac{11\times13}{11\times10} = \frac{13}{10} = 1.3$

4. Show that $7\sqrt{5}$ is irrational. (Reasoning and Proof)

A. Let us assume, the contrary, that is rational. i.e., we can find co-primes a and b (b 0)

such that .

rearranging we get

a and b are integers, $\frac{a}{7b}$ is rational and so is rational.

But this contracdicts the fact that is irrational.

So, we conclude that is irrational.

5. Determine the value of the following. *(Communication)*

i) , ii) $\log_2 512$.

A. i)

$\Rightarrow 2^x = \frac{1}{16}$

[illegible] $\Rightarrow x = -4$

ii) $\log_2 512 = x$ say

$\therefore \log_2 512 = 9$

6. Simplify each of the following expressions as log N. *(problem solving)*

i) 2 log 3 + 3 log 5 – 5 log 2

ii) log 10 + 2 log 3 – log 2.

A. i) 2 log 3 + 3 log 5 – 5 log 2

$= \log 3^2 + \log 5^3 - \log 2^5$ (m log x = log x^m)

= log 9 + log 125 – log 32

= log (9 x 125) – log 32 (log x + log y = log xy)

= log 1125 – log 32

($\because$ log x – log y =)

ii) log 10 + 2 log 3 – log 2

= log 10 + log 3^2 – log 2 (m log x = log x^m)

= log 10 + log 9 – log 2

= log (10 x 9) – log 2 (log x + log y = log xy)

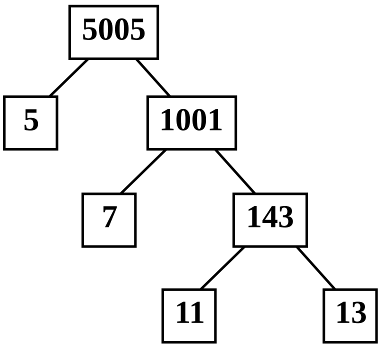
$$\left(\because \log x - \log y = \log \frac{x}{y}\right)$$

= log 45

ONE MARK QUESTIONS

1. Express the number 5005 as a product of its prime factors. (problem solving)

A.



$\therefore$ 5005 = 5 x 7 x 11 x 13

2. Find any rational number between the pair of numbers [illegible] and [illegible]. (connection)

$$= \frac{[illegible]}{2} = \frac{[illegible]}{2} = \frac{\frac{21}{3}}{2}$$

A. We know that the rational number between a and b is

$\therefore$ The rational number between

$$= \frac{7}{2} = 3.5 = 3\frac{1}{2}$$

3. If HCF (306, 657) = 9 then find LCM (306, 657). (problem solving)

A. LCM (306, 657) = x say

LCM x HCF = product of the two numbers

$$x \times 9 = 306 \times 657$$

$\therefore$ LCM (306, 657) = 22358.

4. Write the following in exponential form. (communication)

i) $\log_4 64 = 3$, ii) $\log_a \sqrt{x} = b$

A. i) $\log_4 64 = 3$ $4^3 = 64$

ii)

5. Expand log 15. (communication)

A. log 15 = log(3 x 5) = log 3 + log 5

($\because$ log xy = log x + log y)

6. Show the real number , on the number line. (Visualisation and Representation)

A.

$\frac{-10}{10}$

Divide 1 unit into 10 equal parts

7. Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero ? (Reasoning and Proof)

A. For the number 4^n to end with digit zero for any natural number n. It should be divisible by 5. This means that the prime factorisation of 4^n should contain the prime number 5. But it is not possible because $4^n = (2)^{2n}$. Since 5 is not present in the prime factorisation, so there is no natural number n for which 4^n ends with the digit zero.

$\Rightarrow a^b = \sqrt{x}$

FOUR MARKS QUESTIONS

1. Prove that is irrational. (Reasoning and Proof)

A. Let us assume, to the contrary, that is rational.

So, we can find integers a and b (0)

Such that , where a and be are co-prime. So,

Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2.

Now, by statement it follows that if 2 divides a^2 it also divides a. So we can write a = 2c for some integer C.

Substituting for a, we get $2b^2 = 4c^2$, that is $b^2 = 2c^2$

This means that 2 divides b^2, and so 2 divides b.

Therefore, both a and b have 2 as a common factor.

But this contradicts the fact that a and b are co-prime and have no common factors other than 1.

This contradiction has arisen because of our assumption that is rational. So, we conclude that is irrational.

2. Prove that is irrational. (Reasoning and Proof)

A. Let us assume, to the contrary, that $3+2\sqrt{5}$ is rational.

Then there exist co-prime positive integers a and b such that

Since a and b are integers we get $\frac{a}{b} = -3$ is rational.

So, is rational

But, this contradicts the fact that is irrational. So, our assumption is not correct.

 is irrational.

3. Prove that $\sqrt{3} + \sqrt{5}$ is irrational. *(Reasoning and Proof)*

A. Let us assume, to the contrary, that $\sqrt{3} + \sqrt{5}$ is a rational number.

Then, there exist co-prime positive integers a and b such that

$$\sqrt{3} + \sqrt{5} = \frac{a}{b}$$

[illegible] $2\sqrt{3}\,\frac{a}{b}$

$$\Rightarrow \left(\frac{a}{b} - \sqrt{3}\right)^2 = (\sqrt{5})^2$$

(squaring both sides)

$$\Rightarrow \frac{a^2}{b^2} + 3 - 2\sqrt{3}\,\frac{a}{b} = 5$$

$$\Rightarrow \frac{a^2}{b^2} - 2 = 2\sqrt{3}\,\frac{a}{b}$$

$$\Rightarrow \frac{a^2 - 2b^2}{b^2} \times \frac{b}{a} = 2\sqrt{3}$$

$$\Rightarrow \frac{a^2 - 2b^2}{2ab} = \sqrt{3}$$

Since a, b are integers $\frac{a^2 - 2b^2}{2ab}$ is rational.

and so $\sqrt{3}$ is a rational number

This contradicts the fact that is irrational. So, our assumption is not correct.

Hence, is irrational.

4. Prove the first law of logarithms. *(Reasoning Proof)*

A. The first law of logarithms states

$\log_a xy = \log_a x + \log_a y$

Let $x = a^n$ and $y = a^m$ where $a > 0$ and $a \neq 1$. Then we know that we can write

$\log_a x = n$ and $\log_a y = m$ ....(1)

Using the first law of exponents we know that $a^n.a^m = a^{n+m}$

$x.y = a^n.a^m = a^{n+m}$ i.e., $xy = a^{n+m}$

writing in the logarithmic form, we get

$\log_a xy = n + m$ .... (2)

$\log_a xy = \log_a x + \log_a y$ *(From (1))*

5. Prove the third law of logarithms. *(Reasoning and Proof)*

A. The third law of logarithms states

$\log_a x^m = m.\log_a x$

$\sqrt{3} + \sqrt{5}$

Let $x = a^n$ so $\log_a x = n$ .... (1)

Suppose, we raise both sides of $x = a^n$ to the power m, we get

$x^m = (a^n)^m$

$x^m = a^{nm}$ *(using laws of exponents)*

writing x in the logarithmic form, we get

$\log_a x^m = nm = mn = m.\log_a x$ *(From eq.(1))*

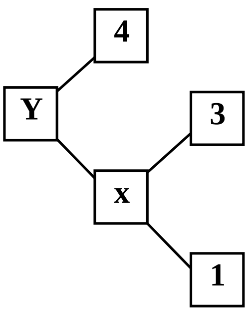
$\therefore \log_a x^m = m.\log_a x$.

OBJECTIVE TYPE QUESTIONS

1. The prime factor of 2 7 11 23+23 is .... (D)

A) 7 B) 11 C) 17 D) 23

2. The values of x and y in the given figure are .... (B)



A) x = 10, y = 14

B) x = 21, y = 84

C) x = 21, y = 25

D) x = 10, y = 40

3. Which of the following is not irrational number. (C)

A) B) C) D)

4. The reciprocal of two irrational numbers is .... (B)

A) always rational no. B) always an irrational number

C) sometimes a rational number, sometimes an irrational number

D) not a real number

5. The decimal expansion of is .... (A)

A) 2.125 B) 2.25 C) 2.375 D) 2.0125

6. is .... (C)

A) An integer B) An irrational C) A rational D) A Prime Number

7. Decimal expansion of number has .... ( )

A) A terminating decimal B) Non-terminating decimal

C) Non terminating, non repeating D) Terminating after two places of decimal

8. If $a = 2^3$ 3, b = 2 3 5, C = 3^n 5 and LCM (a, b, c) = 2^3 3^2 5. Then n = ... (B)

A) 1 B) 2 C) 3 D) 4

9. If n is any natural number, then $6^n - 5^n$ always ends with .... (A)

A) 1 B) 3 C) 5 D) 7

10. If $\log_{10} 2 = 0.3010$ then $\log_{10} 8$ = ... (B)

A) 0.3010 B) 0.9030 C) 2.4080 D) None

Fill up the blanks :

1. If the HCF of two numbers is 1, then the two numbers are called .... (co-prime)
2. If two positive numbers a and b are written as $a = x^5y^2$, $b = x^3y^3$, where x and y are prime numbers then the HCF (a, b) = ..., LCM (a, b) = ... (x^3y^2, x^5y^3)
3. If p, q are primes then is .... (irrational)
4. $\log a^P.b^q$ = ... (P log a + q log b)
5. If x and y are prime numbers, then HCF of (x, y) = ... (1)
6. The power of 2 in the prime factorisation of 4000 is .... (5)
7. What is the least number that multiplied with $\sqrt{18}$ to get a irrational number. ( )
8. After how many digits will be decimal expansion of to an end ? .... (3)
9. If $\log_3 27 = x$ then x = ... (3)
10. 0.0875 = ... (Write in the form of $2^n \times 5^m$)

CHAPTER - 2
SETS

- In Mathematics, Set Theory was developed by George Cantor (1845 – 1918)
- **Set :** A well defined collection of objects is called a Set.

  'Well defined' means that

  i) All the objects in the set should have a commong feature or property : and

  ii) It should be possible to decide whether any given object belongs to the set or not.
- We usually denote a set by capital letters and the elements of a set are represented by small letters.

  Ex : Set of Vowels in English language V = {a,e,i,o,u}

  Set of even numbers, E = {2, 4, 6, 8, ...}

  Set of odd numbers, O = {1,3,5,7,11,13,........}

  Set of Prime Numbers, P = {2,3,5,7,11,13,...........}
- Any element or object belonging to a set, then we use symbol ' ' (beliongs to), if it is not belonging to it is denoted by the symbol ' ' (does not belongs to)

  Ex : In natural numbers set N, 1   N and 0   N.

  Roaster Form : All elements are written in order by separating commas and are enclosed with in curly brackets is called Roaster form. In this form elements should not repeated.

  Ex : Set of prime numbers less than 13 is P = {2,3,5,7,11}

  Set Builder Form : In set builder form, we use a symbol x (or any other symbol y, z etc.) for the element of the set. This is followed by a colon (or a vertical line), after which we write the characteristic property possessed by the elements of the set. The whole is enclosed within curly brackets.

  Ex : P = {2,3,5,7,11}. This is the set of all Prime Numbers less than 13. It can be represented in the set builder form as

  P = {x ; x is a prime number less than 13}

  (or)

  P = {x/x is a prime number less than 13}

  Null Set : A set which does not contain any element is called the empty set or the null set or a void set. It is denoted by ϕ or { }.

  Ex : A = {x/1 < x < 2, x is a natural number}

  B = {x/x^2 – 2 = 0 and x is a rational number}

  Finite Set : A set is called a finite set if it is possible to count the number of elements in it.

  Ex : A = {x ; x   N and (x – 1) (x – 2) = 0} = {1,2}

  B = {x ; x is a day in a week} = {SUN, MON, TUE, WED, THU, FRI, SAT}

  Infinite Set : A set is called an infinite set if the number of elements in it is not finite (i.e.,) we cannot count the number of elements in it.

  Ex : A = {x/x   N and x is an odd number}

  = {1,3,5,7,9,11,...........}

  B = {x/x is a point on a straight line}

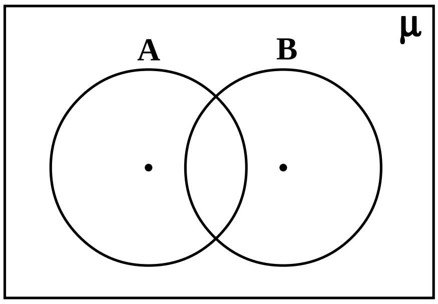
Cardinal Number : The number of elements in a set is called the cardinal number of the set. If 'A' is a set then n(A) represents cardinal number.

Ex : If A = {a,e,i,o,u} then n(A) = 5

If B = {x ; x is a letter in the word INDIA} then n(B) = 4

n(ϕ) = 0

Universal Set : Universal Set is denoted by 'μ' or 'U'. Generally, Universal Set represented by rectangle.



Sub Set : If every element of a set A is also an element of set B, then the set A is said to be a subset of set B. It is represented as A B.

Ex : If A = {4,8,12} ; B = {2,4,6,8,10,12,14} then A is a subset of B (i.e., A B)

- Every set is a subset of itself (A A)
- Empty set is a subset of every set (ϕ A)
- If A B and B C then A C *(Transitive Property)*

Equal Sets : Two sets A and B are said to be 'equal' if every element in A belongs to B and every element in B belongs to A. If A and B are equal sets, then we write A = B.

Ex : The set of prime numbers less than 6, A = {2,3,5}.

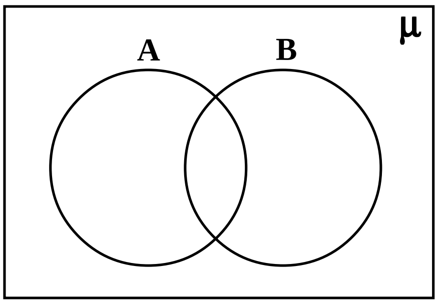
The prime factors of 30, B = {2,3,5}

Since the elements of A are the same as the elements of B, therefore, A and B are equal.

- A B and B A A = B *(Antisymmetric Property)*

Venn Diagrams : Venn - Euler diagram or simply Venn diagram is a way of representing the relationships between sets. These diagrams consist of rectangles and closed curves usually circles.

Ex :



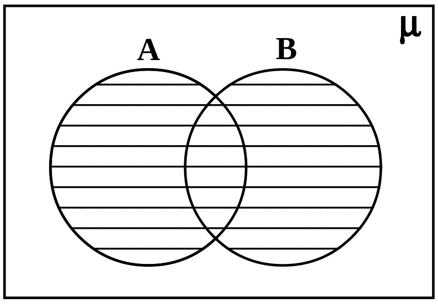
Basic Operations on Sets : We know that arithmetic has operations of additions, subtraction and multiplication of numbers. Similarly in sets, we define the operation of 'union', intersection and difference of sets.

Union of Sets : The union of A and B is the set which contains all the elements of A and also the elements of B and the common element being taken only once. The symbol 'U' is used to denote the union. Symbolically, we write A B and read as 'A union B'.

A   B = {x/x   A or x   B}

Ex : A = {1,2,3,4,5}, B = {2,4,6,8,10}

then A   B = {1,2,3,4,5,6,8,10}



represents A   B

- A   A = A *(Idempotant Law)*
- A   ϕ = A = ϕ   A *(Identity Property)*
- A   μ = μ = μ   A

= If A   B then A   B = B

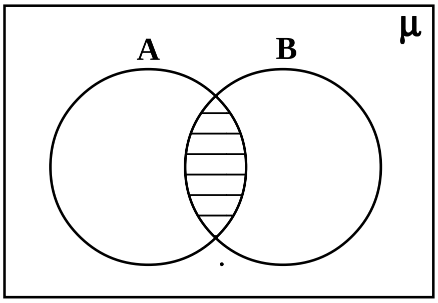
- A   B = B   A *(Commutative Property)*

Intersection of Sets : The intersection of A and B is the set in which the elements that are common to both A and B. The symbol '   ' is used to denote the 'intersection'. Symbolically we write A   B and read as "A intersection B".

A   B = {x/x   A and x   B}

Ex : A = {1,2,3,4,5} and B = {2,4,6,8,10}

then A   B = {2,4}



represents A   B

- A   A = A
- A   ϕ = ϕ = ϕ   A
- A   μ = A = μ   A *(Identity Property)*
- If A   B then A   B = A

∴ A   B = B   A *(Commutative Property)*

Disjoint Sets : If there are no common elements in A and B then the sets are known as disjoint sets.

If A, B are disjoing sets then A   B = f

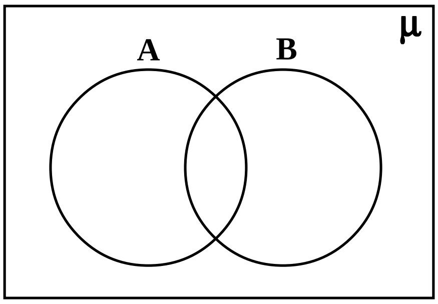
If A   B = f then n(A   B) = 0

Ex : A = {1,3,5,7,.....} ; B = }2,4,6,8,......}

Here A and B have no common elements

∴ A and B are called disjoint sets.

i.e., A   B = ϕ

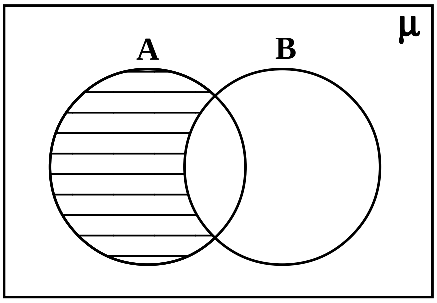


Difference of Sets : The difference of sets A and B is the set of elements which belongs to A but donot belong to B. We denote the difference of A and B by A – B or simply "A minus B".

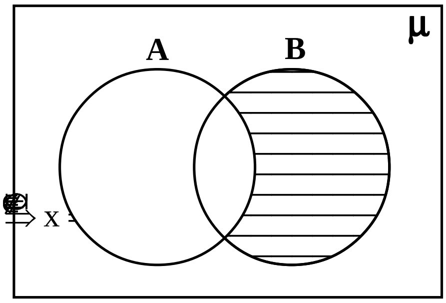
A – B = {x/x   A and x   B}

B – A = {x/x   B and x   A}

Ex : If A = {1,2,3,4,5} and B = {4,5,6,7} then A – B = {1,2,3}, B – A = {6,7}



represents 'A - B'



represents B - A

- A – B   B – A
- A – B, B – A and A   B are disjoint sets.
- n(A   B) = n(A) + n(B) – n(A   B)
- If A, B are disjoint sets then n(A   B) = n(A) + n(B)

TWO MARKS QUESTIONS

1. Which of the following are sets ? Justify your answer? (Reasoning and Proof)

   i) The collection of all the months of a year beginning with the letter 'J'.

   ii) x is an integer and $x^2 = 4$.

A. i) All the months of a year begining with the letter 'J' are January, June, July.

   It is a set = {January, June, July}

   ii) $x^2 = 4$

   +2, –2 both are integers. So, it is a set. = {–2, 2}

2. State whether the following statements are true (or) false. (Reasoning and Proof)

   i) 5   {Prime Numbers}

ii) , where Z is the set of integers.

A. i) 5 {Prime Numbers} – False

Because 5 is a prime number.

ii) , where 'Z' is the set of integers. – False. Because is a Rational Number.

3. Let A be the set of prime numbers less than 6 and P the set of prime factors of 30. Check if A and P are equal. (Reasoning and Proof)

A. The set of Prime numbers less than 6, A = {2,3,5}

The prime factors of 30 are 2, 3 and 5.

So, P = {2,3,5}

Since the elements of A are the same as the elements of P, therefore, A and P are equal.

4. Let A = {1,2,3,4,5}, B = {4,5,6,7}. Find A – B and B – A. Are they equal ? (Reasoning and Proof)

A. A = {1,2,3,4,5}, B = {4,5,6,7}

A – B = {1,2,3,4,5} – {4,5,6,7} = {1,2,3}

B – A = {4,5,6,7} – {1,2,3,4,5} = {6,7}

Note that A – B B – A.

5. Which of the following are infinite or infinite. (Reasoning and Proof)

i) A = {x : x N and (x – 1) (x – 2) = 0}

$\frac{8}{11} \in Z$

ii) B = {x : x is a line which is parallel to the x-axis}

A. i) Given Set A = {x : x N and (x – 1) (x – 2) = 0}

from (x – 1) (x – 2) = 0

x = 1 or x = 2

∴ A = {1, 2} It is a finite set.

ii) B = {x : x is a line which is parallel to the x-axis}

We cannot say the no. of elements of this set. So, it is infinite set.

FOUR MARK QUESTIONS

1. Write the following sets in the set-builder form. (Connection)

i) A = {1,2,3,4,5} ii) B = {5,25,125,625} iii) C = {1,2,3,6,7,14,21,42} iv) D = {1,4,9,.....100}

A. Set-builder form of the given sets

i) A = {x : x is a natural number, x < 6}

ii) B = {5^x : x N, x 4}

iii) C = {x : x is a natural number which divides 42}

iv) D = {x^2 : x in square of natural number and not greater than 10}

2. If A = {3,4,5,6,7} and B = {1,6,7,8,9}. Then find i) A B ii) A B iii) A – B iv) B – A. (Problem Solving)

A. A = {3, 4, 5, 6, 7} ; B = {1,6,7,8,9}

i) A B = {3,4,5,6,7} {1,6,7,8,9} = {1,3,4,5,6,7,8,9}

ii) A B = {3,4,5,6,7} {1,6,7,8,9} = {6,7}

iii) A – B = {3,4,5,6,7} – {1,6,7,8,9} = {3,4,5}

iv) B – A = {1,6,7,8,9} – {3,4,5,6,7} = {1,8,9}

3. If A = {2,3,5} then find A ϕ and f A and compare. (problem solving)

A. A = {2,3,5}

A ϕ = {2,3,5} { } = {2,3,5} = A

ϕ A = { } {2,3,5} = {2,3,5} = A

∴ A ϕ = ϕ A = A

4. If A = {x : x is a natural number}, B = {x : x is an even natural number}, C = {x : x is an odd natural number}, D = {x : x is a prime number}

Find A B, A C, A D, B C, B D, C D. (problem solving)

A. A = {x : x is a natural number} = {1,2,3,4,5,.......}

B = {x : x is an even natural number} = {2,4,6,8,.....}

C = {x : x is an odd natural number} = {1,3,5,7,.....}

D = {x : x is a prime number} = {2,3,5,7,........}

A B = {1,2,3,4,5,....} {2,4,6,8,........}

= {2,4,6,.......} = Even Natural Set

A C = {1,2,3,4,5,....} {1,3,5,7,....}

= {1,3,5,7,.....} = odd natural set.

A D = {1,2,3,4,5,....} {2,3,5,7,....}

= {2,3,5,7,....} = prime natural set

B C = {2,4,6,8,....} {1,3,5,7,....} = ϕ.

B D = {2,4,6,8,....} {2,3,5,7,....}

= {2} = even prime set.

C D = {1,3,5,7,....} {2,3,5,7,....}

= {3,5,7,....} = odd prime set.

5. If A = {1,2,3,4,5}, B = {4,5,6,7} The sets A – B, B – A and A B are naturally disjoint sets. (problem solving)

A. A = {1,2,3,4,5} – {4,5,6,7}

A – B = {1,2,3,4,5} – {4,5,6,7} = {1,2,3}

B – A = {4,5,6,7} – {1,2,3,4,5} = {6,7}

A B = {1,2,3,4,5} {4,5,6,7} = {4,5}

∴ A – B, B – A and A B are disjoint sets.

ONE MARK QUESTIONS

1. A = {x : x is a prime number which is a divisor of 60}. Write the set in roster form. (Connection)

A. A = {2,3,5}

2. If A = {x,y,z}. How many subsets does the set A have ? (problem solving)

A. A = {x,y,z}

The subsets of A are

ϕ, {x}, {y}, {z}, {x, y}, {y, z}, {x, z}, {x,y,z}

The no. of subsets are = 8.

3. If A = {1,2,3,4,5,6}, B = {2,4,6,8,10} then find n(A B). (problem solving)

A. A = {1,2,3,4,5}, B = {2,4,6,8,10}

A B = {1,2,3,4,5,6,8,10}

n(A B) = 8

4. If A = {5,6,7,8}, B = {7,8,9,10} then find A B. (problem solving)

A. A B = {5,6,7,8} {7,8,9,10} = {7,8}

5. n(A) = 5, n(B) = 3, n(A B) = 2 then find n(A B). (problem solving)

A. n(A B) = n(A) + n(B) – n(A B)

= 5 + 3 – 2

∴ n(A B) = 6

6. If A = {0,2,4} then find A A, A A. (problem solving).

A. A = {0,2,4}

A A = {0,2,4} {0,2,4} = {0,2,4} = A

A A = {0,2,4} {0,2,4} = {0,2,4} = A

7. Give an example of disjoint sets. (Connection)

A. A = {2,4,6,8,....} ; B = {1,3,5,7,....}

A and B are disjoint sets.

8. Give an example for a null set. (Connection)

A. A = {x : x is an integer between 2 and 3}

9. By giving examples verify that if A, B are disjoint sets then A B is a null set. (Connection).

A. Examples for disjoint sets A = {1,3,5,6}, B = {2,4,6,8}

A B = {1,3,5,7} {2,4,6,8} = ϕ

We observe that A B is a null set.

10. By giving examples verify that if A. B are disjoint sets then n(A B) = n(A) + n(B). (Connection)

A. Examples for disjoint sets

A = {1,3,5,7}, B = {2,4,6,8}

n(A) = 4, n(B) = 4

A B = {1,3,5,7} {2,4,6,8} = {1,2,3,4,5,6,7,8}

n(A B) = 8

A B = ϕ { A, B are disjoint sets)

n(A B) = 0

n(A) + n(B) = 4 + 4 = 8 = n(A B)

∴ n(A B) = n(A) + n(B).

OBJECTIVE QUESTIONS

1. Which of the following set is not null set ? (D)

A) {x : 1 < x < 2, x is a natural number} B) {x : x^2 – 2 = 0 and x Q}

C) {x : x^2 = 4 and x is odd} D) {x : x is a prime number divisible by 2}

2. If A = {a,b,c}, the number of subsets of A is .... (C)

A) 3 B) 4 C) 8 D) 12

3. For every set A, A   ϕ = ... (B)

A) A B) ϕ C) m D) A – ϕ

4. Two sets A and B are said to be disjoint if .... (D)

A) A – B = ϕ B) A   B = ϕ C) A   B = B   A D) A   B = ϕ

5. n(A   B) = ... (D)

A) n(A) – n(B) B) n(A) + n(B)

C) n(A) + n(B) +n(A   B) D) n(A) + n(B) – n(A   B)

6. If A = {1,2,3,4,5} then the Cardinal number of A is .... (B)

A) 2^5 B) 5 C) 4 D) 5^2

7. (A – B)   (B – A) = ... (C)

A) A B) B C) ϕ D) μ

8. Set builder form of A   B = ... (B)

A) {x : x   A and x   B} B) {x : x   A or x   B}

C) {x : x   A and x   B} D) {x : x   A and x   B}

9. If A   B, then A   B = ... (A)

A) A B) B C) ϕ D) A   B

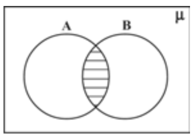
10. Which is true ? (D)

A) Symbol of Null Set is f B) Symbol of universal set is m

C) Symbol of subset is D) All the above

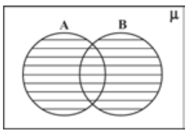
FILL UP THE BLANKS

1. The shaded region



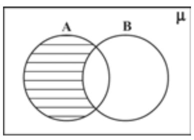
in the adjacent figure is .... (A   B)

2. The shaded region



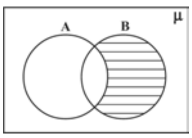
in the adjacent figure is .... (A   B)

3. The shaded region in



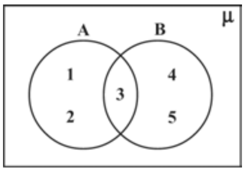
the adjacent figure is .... (A – B)

4. The shaded region in



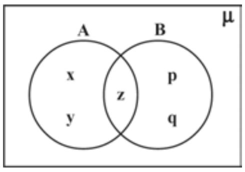
the adjacent figure is .... (B – A)

5. From the figure



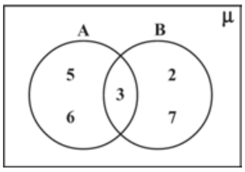
A   B = ...   ({3})

6. From the figure



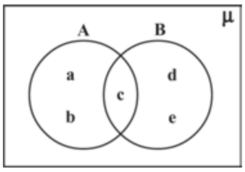
A   B = ...   ({x,y,z,p,q})

7. From the figure



(A – B)   (B – A) = ...   ({2,5,6,7})

8. From the figure



(A – B)   (B – A) = ...   (ϕ)

9. Set builder form of A   B = ...   ({x : x   A and x   B})

10. Set builder form of A – B = ...   ({x : x   A and x   B})

CHAPTER - 3
POLYNOMIALS

- This chapter will covered from Group - A in the Sections of I and III.
- 5 Marks will covered under Section - IV.
- Marks weightage (Max. 15 Marks) has shown below :

1	x	2	=	2
1	x	1	=	1
1	x	4	=	4
1	x	5	=	5
6	x	$^1/_2$	=	3

- **Definition :** Polynomials are algebraic expression constructed using constants and variables.

  Ex : $2x + 5$; $3x^2 - 7x + 8$; $-9y + 8$; x^4 are some polynomials.

  are not polynomials.

- **Degree of Polynomial :** If P(x) is a polynomial in x, the highest power of x in P(x) is called the degree of the polynomial P(x).

  Ex : 1) Degree of a polynomial $P(x) = 7x - 8$ is 1 (one).

  A polynomial of degree 1 (one) is called a linear polynomial.

  2) A polynomial of degree 2 is called a quadratic polynomial.

  Ex : $P(x) = x^2 + 5x + 4$; $-2x^2 - 3x + 2$ $\frac{3}{x^2}, \frac{1}{\sqrt{7}x}, \frac{7}{y+9}, \sqrt{5x^3}$

  3) A polynomial of degree 3 is called a cubic polynomial.

  Ex : $3x^3 - 4x^2 + 5x + 7$; $2 - x^3$, $y^3 - 3y + \sqrt{7}$

  Value of a Polynomial : If P(x) is a polynomial in x, and if K is a real number, then the value obtained by replacing x by K in P(x), is called the value of P(x) at x = K is denoted by P(K).

Examples :

1. **If $P(x) = 3x^2 - 2x + 5$, find the values of P(1), P(2), P(0), P(–1), P(–2).**

Sol. Let $P(x) = 3x^2 - 2x + 5$

we where $P(1) = 3(1)^2 - 2(1) + 5 = 3 - 2 + 5 = 6$

Also $P(2) = 3(2)^2 - 2(2) + 5 = 3(4) - 4 + 5 = 13$

$P(0) = 3(0)^2 - 2(0) + 5 = 0 - 0 + 5 = 5$

$P(-1) = 3(-1)^2 - 2(-1) + 5 = 3 + 2 + 5 = 10$

$P(-2) = 3(-2)^2 - 2(-2) + 5 = 12 + 4 + 5 = 21$

2. **Let $P(x) = x^2 - 4x + 3$. Find the values of P(0), P(1), P(2), P(3) and obtain zeroes of the polynomial P(x).**

Sol. Let $P(x) = x^2 - 4x + 3$

$P(0) = (0)^2 - 4(0) + 3 = 3$

$P(1) = (1)^2 - 4(1) + 3 = 4 - 4 = 0$

$P(2) = (2)^2 - 4(2) + 3 = 4 - 8 + 3 = 7 - 8 = -1$

$P(3) = (3)^2 - 4(3) + 3 = 9 - 12 + 3 = 12 - 12 = 0$

∴ As P(1) = 0, and P(3) = 0 ; 1 and 3 are said to be zeroes of the polynomial P(x).

Relationship between zeroes and coefficients of a polynomial :

i) **Quadratic Polynomial :** General form of quadratic polynomial in x : $P(x) = ax^2 + bx + c$ $(a \neq 0)$

Let the zeroes of P(x) are α, β

Sum of the zeroes

Product of the zeroes

ii) **Cubic Polynomial :** General form of cubit polynomial in x = $P(x) = ax^3 + bx^2 + cx + d$

Let α, β, γ are three zeroes of cubic polynomial P(x)

We see relationship between α, β, γ and a, b, c, d.

Sum of its zeroes (α + β + γ)

Sum of the products of the zeroes taken two at a time :

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{\text{coefficient of } x}{\text{coefficient of } x^3} = \left(\frac{c}{a}\right)$$

Product of its zeroes

Quadratic Polynomial (if its zeroes are given) : $K[x^2 - x(\alpha + \beta) + \alpha\beta]$ where K is a constant.

2 MARKS QUESTIONS

1. If $P(x) = 5x^7 + 6x^5 + 7x - 6$, find (i) coefficient of x^7, (ii) degree of P(x) (iii) constant term (iv) coefficient of x^7.
2. If $P(t) = t^3 - 1$, find the values of P(1), P(–1), P(0), P(2), P(–).
3. Check whether 3 and –2 are the zeroes of the polynomial P(x), when $P(x) = x^2 - x - 6$.
4. Find the zeroes of the polynomial $P(x) = x^2 + 5x + 6$.
5. Why are $^1/_4$ and –1 zeroes of the polynomials $P(x) = 4x^2 + 3x - 1$.
6. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.
7. Find a quadratic polynomial if the zeroes of it are 2 and $^{-1}/_3$ respectively.
8. Find the quadratic polynomial whose sum of zeroes is $^1/_4$ and the product of its zeroes is –1 ?
9. Divide the polynomial $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$ and find the quotient and remainder.

1 MARK QUESTIONS

10. Find the number of zeroes of the polynomials (i) $P(y) = y^2 - 1$, (ii) $q(z) = z^3$ and also find zeroes.

11. Find the zeroes of $P(x) = (x + 2)(x + 3)$.

12. Find the zeroes of cubic polynomials (i) $x^2 - x^3$, (ii) $x^3 - 4x$.

13. Define Euclid's division algorithm.

14. Give examples of polynomials P(x), g(x), q(x) and r(x) which satisfy the division algorithm and

    (i) deg P(x) = deg q(x), (ii) deg q(x) = deg r(x), (iii) deg r(x) = 0.

15. Write one polynomial that has one zero if P(x) is quadratic polynomial.

4 MARKS QUESTIONS

1. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are and .

2. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$ respecitvely. Find g(x) ?

3. Verify that 3, –1, $^{-1}/_{3}$ are the zeroes of the cubic polynomial $P(x) = 3x^3 - 5x^2 - 11x - 3$, and then verify the relationship between the zeroes and the coefficients.

4. If α, β, γ are the zeroes of the given cubic polynomials, find the values as given in the table.

S.No.	Cubic Polynomial	$\alpha+\beta+\gamma$ $\sqrt{2}$	$\alpha\beta+\beta\gamma+\gamma\alpha$	$\alpha\beta\gamma$
1.	$x^3 + 3x^2 - x - 2$			
2.	$4x^3 + 8x^2 - 6x - 2$			
3.	$x^3 + 4x^2 - 5x - 2$			
4.	$x^3 + 5x^2 + 4$			

5 MARKS QUESTIONS (GRAPH)

1. Draw the graphs of the quadratic polynomial and find the zeroes. Justify the answers.

   (i) $y = x^2 - 3x - 4$, (ii) $y = x^2 - 6x + 9$, (iii) $P(x) = x^2 - 4x + 5$, (iv) $P(x) = x^2 + 3x - 4$, (iv) $P(x) = x^2 - x - 12$.

BITS (PART - B)

FILL IN THE BLANKS

1. Coefficient of 'x' in the polynomial $x^4 - 7x^2 + 9$ is ....
2. The number of zeroes of the polynomial $P(y) = y^2 - 9$ is .... and they are ....
3. A quadratic polynomial, the sum of whose zeroes is 0 and one zero is 2 ; is ....
4. If α, β are the zeroes of $x^2 + 7x + 10$, then $\alpha\beta$ = ...
5. The degree of a constant term in a polynomial is ....
6. The zero of the linear polynomial px + q is ....
7. If one zero of the polynomial $x^2 - kx - 4$ is –1, then the value of K is ....

8. The quadratic polynomial has atmost .... zeroes.
9. A real number K is a .... of f(x), if f(k) = 0.
10. The graph of the polynomial $y = ax^2 + bx + c$ is an upward parabola if 'a' is ....
11. If the graph of a polynomial does not intersect the x-axis, then the number of zeroes of the polynomial is ....
12. If two zeroes of the polynomial $ax^3 + bx^2 + cx + d$ are each equal to zero, then the third zero is ....

MATCH THE FOLLOWING :

I.

	GROUP - A		**GROUP - B**
1.	constant polynomial	[ ]	A) $P(x) = ax^2 + bx + c$, $(a \neq 0)$
2.	linear polynomial	[ ]	B) $P(y) = ay^4 + by^3 + cy^2 + dy + e$ (a 0)
3.	quadratic polynomial	[ ]	C) $q(x) = ax + b$, (a 0)
4.	cubic polynomial	[ ]	D) $P(t) = at^3 + bt^2 + ct + d$ (a 0)
5.	biquadratic polynomial	[ ]	E) $P(z) = a$, ('a' is a constant)
			F) $P(x) = x^5$

II. Given that the polynomial $P(x) = x^4 - x^3 - 5x^2 - 2x + 12$.

	Group - A		**Group - B**
1.	Sum of the coefficients	[ ]	A) 4
2.	Coefficient of x^0	[ ]	B) –6
3.	Degree of the polynomial	[ ]	C) 5
4.	Sum of the coefficient of x^3 and x^2	[ ] $\neq$	D) 3
5.	No. of zeroes atmost	[ ]	E) 6
			F) 12

III. Graph of the curve, the points at which it cuts the x-axis.

	Group - A		**Group - B**
1.	$y = x^3$	[ ]	A) (2, 0)
2.	$y = x^3 - 4x$	[ ]	B) (4, 0) (–4, 0)
3.	$y = x^2 - 4x + 4$	[ ]	C) (3, 0) (–2, 0)
4.	$y = x^2 - 16$	[ ]	D) (0, 0) (1, 0)
5.	$y = x^2 - x - 6$	[ ]	E) (0, 0)
			F) (0, 0), (2, 0), (–2, 0)

ANSWERS (2 MARKS)

1. Let $P(x) = 5x^7 - 6x^5 + 7x - 6$

   i) Coefficient of $x^5 = -6$

   ii) Degree of P(x) = 7

   iii) Constant term = –6

   iv) Coefficient of $x^7 = 5$

2. Let $P(t) = t^3 - 1$ (given)

$P(1) = (1)^3 - 1 = 1-1 = 0$

$P(-1) = (-1)^3 - 1 = -1-1 = -2$

$P(0) = (0)^3 - 1 = 0-1 = -1$

$P(2) = (2)^3 - 1 = 8-1 = 7$

$P(-2) = (-2)^3-1 = -8-1 = -9$

3. We know that a real number K is said to be a zero of a polynomial P(x) if P(K) = 0

   Let $P(x) = x^2 - x - 6$

   $P(3) = (3)^2 - 3 - 6 = 9-3-6 = 9-9 = 0$

   $P(-2) = (-2)^2-2-6 = 4+2-6 = 6-6 = 0$

   ∴ 3 and –2 are the zeroes of the polynomial $P(x) = x^2-x-6$.

4. Let $P(x) = 1x^2 + 5x + 6$

   $= 1x^2 + 3x + 2x + 6$

   $= x(x+3) + 2(x+3)$

   $= (x+3)(x+2)$

   To find zeroes ; P(x) = 0 (x+3) (x+2) = 0

   x+3 = 0 (or) x+2 = 0

   x = –3 x = –2

   ∴ The zeroes of $x^2 + 5x + 6$ are –2 and –3.

5. Given that $P(x) = 4x^2 + 3x - 1$

   $$P\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) - 1$$

   $$= 4\left(\frac{1}{16}\right) + \frac{3}{4} - 1$$

   $$= \frac{1}{4} + \frac{3}{4} - 1$$

   $$= \frac{0}{4} = 0$$

   $P(-1) = 4(-1)^2 + 3(-1)$

   = 4 – 3 – 1

   = 4 – 4

   = 0

   Since and P(–1) are each equal to zero, and –1 are the zeroes of the polynomial P(x).

6. To find zeroes of the polynomial $P(x) = x^2 - 3 = 0$

$x^2 = 3$

$\therefore$ The zeroes of $x^2 - 3$ are and – .

Verificiation : Sum of the zeroes = – = 0

($\because 1x^2 - 0.x - 3 = P(x)$)

Product of the zeroes = ( ) (–) = –( )^2 .

7. Let a, b be the zeroes of the quadratic polynomial

$P(x) = ax^2 + bx + c, (a \neq 0)$

Here $\alpha = 2$,

Sum of the zeroes :

Product of the zeroes :

$\therefore$ The required quadratic polynomial will be [illegible]

(where K is a constant)

$$= K\left[x^2 - x\left(\frac{5}{3}\right) - \frac{2}{3}\right]$$

when K = 3, the quadratic polynomial will be

8. Let a, b be the zeroes of the quadratic polynomial.

Sum of the zeroes = $= (\alpha + \beta) = \frac{1}{4}$

Product of the zeroes

$\therefore$ The required quadratic polynomial will be where K is a constant.

when K = 4, the quadratic polynomial will be .

9. Let $P(x) = x^3 - 3x^2 + 5x - 3$ as Dividend and $g(x) = x^2 - 2$ as Divisor.

The given polynomial is in standard form.

$x^2 - 2)\ x^3 - 3x^2 + 5x - 3\ (x - 3$

$x^3 + 0x^2 - 2x$

....

$-3x^2 + 7x - 3$

$-3x^2 + 0x + 6$

....

$7x - 9$

$\therefore$ We stop here since the degree of $(7x - 9) <$ degree of $(x^2 - 2)$.

So, the quotient is $(x - 3)$ and the remainder $= 7x - 9$.

ANSWERS (1 MARK)

1.i) Let $p(y) = y^2 - 1$ is a quadratic polynomial

It has atmost two zeroes

$$4\left[K\left[\frac{4x^2 - x - 4}{4}\right]\right] = 4x^2 - x - 4$$

To find zeroes, Let $p(y) = 0$

$\Rightarrow y^2 - 1 = 0$

$y^2 = 1$

y=

$y = 1$ (or) -1

$\therefore$ The zeroes of the polynomial are 1 or –1.

ii) Let $q(z) = z^3$ and it is a cubic (3rd degree) polynomial. It has atmost three zeroes.

Let $q(z) = 0$

$z^3 = 0$

$z = 0$

$\therefore$ The zero of the polynomial = 0.

11. To find zeroes of p(x), Let $p(x) = 0$

$(x + 2)(x + 3) = 0$

$x + 2 = 0$ (or) $x + 3 = 0$

$x = -2 \quad x = -3$

So, the zeroes of the polynomial are –2 and –3.

12.i) To find zeroes of given polynomial : $x^2 - x^3 = 0$

$x^2(1 - x) = 0$

$x^2 = 0$ (or) $1 - x = 0$

$x = 0 \quad +x = +1$

Zeroes of cubic polynomial are '0' and '1'.

ii) To find zeroes of given polynomial : $x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0$ (or) $x^2 - 4 = 0$

$x^2 - 2^2 = 0$

$(x + 2)(x - 2) = 0$

$x + 2 = 0$ (or) $x - 2 = 0$

$x = -2$ (or) $x = 2$

$\therefore$ Zeroes of cubic polynomial are 0, –2 and 2.

13. If p(x) and g(x) are any two polynomial with g(x)   0, then we can find polynomial q(x) and r(x) such that p(x) = g(x) x q(x) + r(x)

where either r(x) = 0 or degree of r(x) < degree of g(x).

This result is known as Euclid's Division Algorithm for Polynomials.

14. Examples of polynomial p(x), g(x), q(x) and r(x) which satisfy the division algorithm.

i) $p(x) = 4x^2 - 6x + 4$

$g(x) = 2$, $q(x) = 2x^2 - 3x + 2$, $r(x) = 0$

for deg p(x) = deg q(x).

ii) $p(x) = x^3 + 2x^2 + x - 6$

$g(x) = x^2 + 2$, $q(x) = x + 2$, $r(x) = -x - 10$

for deg q(x) = deg r(x). $\neq\sqrt{2}$

iii) $p(x) = x^3 + 5x^2 - 3x - 10$

$g(x) = x^2 - 3$, $q(x) = x + 5$, $r(x) = 5$.

for deg r(x) = 0

ANSWERS FOR 4 MARKS

1. The given polynomial is $2x^4 - 3x^3 - 3x^2 + 6x - 2$.

Two of its zeroes are   and –

$\therefore$ (x–   ) (x+   ) = $(x^2 - 2)$ is a factor of the given polynomial.

Now, we apply the division algorithm to the given polynomial and $x^2 - 2$

$x^2 - 2)\ 2x^4 - 3x^3 - 3x^2 + 6x - 2\ (2x^2 - 3x + 1$

$2x^4 + 0 - 4x^2$

....

$-3x^3 + x^2 + 6x$

$-3x^3 + 0 + 6x$

....

$x^2 - 2$

$x^2 - 2$

....

0

So, $2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$

Now we factorize : $2x^2 - 3x + 1$

$= 2x^2 - 2x - 1x + 1$

$= 2x(x - 1) - 1(x - 1)$

$= (x - 1)(2x - 1)$

So its zeroes are $x - 1 = 0$ (or) $2x - 1 = 0$

$x = 1$ or $2x = 1$

$x = 1$ or $x = {}^{1}/_{2}$.

$\therefore$ The zeroes of the given polynomial are .

2. Let $p(x) = x^3 - 3x^2 + x + 2$ as Dividend

$g(x)$ as Divisor,

$q(x) = x - 2$ as quotient

$r(x) = -2x + 4$ as remainder

By division algorithm, we have

$p(x) = g(x) \times q(x) + r(x)$

$g(x)\ \ q(x) = p(x) - r(x)$

$g(x)\ \ (x - 2) = x^3 - 3x^2 + x - 2 - (-2x + 4)$

$= x^3 - 3x^2 + x + 2 + 2x - 4$

$= x^3 - 3x^2 + 3x - 2$

$\therefore g(x) = (x^3 - 3x^2 + 3x - 2)\ \ (x - 2)$

$\therefore \sqrt{2}, -\sqrt{2}, 1$ and ${}^{1}/_{2}$

$p\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right)^3 - 5\left(\frac{-1}{3}\right)^2 - 11\left(\frac{-1}{3}\right) - 3$

$x - 2)\ 1x^3 - 3x^2 + 3x - 2\,(x^2 - x + 1$

$1x^3 - 2x^2$

....

$-x^2 + 3x$

$-x^2 + 2x$

....

$x - 2$

$x - 2$

....

0

$\therefore g(x) = x^2 - x + 1$

3. Let $p(x) = 3x^3 - 5x^2 - 11x - 3$

$p(3) = 3(3)^3 - 5(3)^2 - 11(3) - 3 = 81 - 45 - 33 - 3 = 81 - 81 = 0$

$p(-1) = 3(-1)^3 - 5(-1)^2 - 11(-1) - 3 = -3 - 5 + 11 - 3 = -11 + 11 = 0$

$$= \frac{-1-5+33-27}{9} = \frac{-33+33}{9} = \frac{0}{9} = 0$$

$\therefore$ 3, –1 and are the zeroes of $3x^3 - 5x^2 - 11x - 3$.

ii) **Verification of Relationship between the zeroes and the coefficients :**

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get

a = 3, b = –5, c = –11, d = –3

and we take zeroes as $\alpha = 3$, $\beta = -1$ and

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3(-1) + (-1)\left(\frac{-1}{3}\right) + \left(-\frac{1}{3}\right)3 = \frac{-9+1-3}{3} = \frac{-11}{3} = \frac{c}{a}$$

$$\alpha\beta\gamma = 3(-1)\left(-\frac{1}{3}\right) = 1 = \frac{-(-3)}{3} = -\frac{d}{a}$$

4.i) The given polynomial is $1x^3 + 3x^2 - 1x - 2$

$$= \frac{9-3-1}{3} = \frac{5}{3} = \frac{-(-5)}{3} = -\frac{b}{a}$$

Comparing this with $ax^3 + bx^2 + cx + d$, we get

a = 1, b = 3, c = –1, d = –2

$$\therefore\ \alpha + \beta + \gamma = -\frac{b}{a} = \frac{-3}{1} = -3$$

ii. The given polynomial is $4x^3 + 8x^2 - 6x - 2$

Comparing this with $ax^3 + bx^2 + cx + d$, we get

a = 4, b = 8, c = –6, d = –2

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(-2)}{4} = \frac{1}{2}$$

iii. By solving as above, we get $\alpha + \beta + \gamma = -4$, $\alpha\beta + \beta\gamma + \gamma\alpha = -5$ and $\alpha\beta\gamma = 2$.

iv. By solving as above, we get $\alpha + \beta + \gamma = -5$, $\alpha\beta + \beta\gamma + \gamma\alpha = 0$ and $\alpha\beta\gamma = -4$

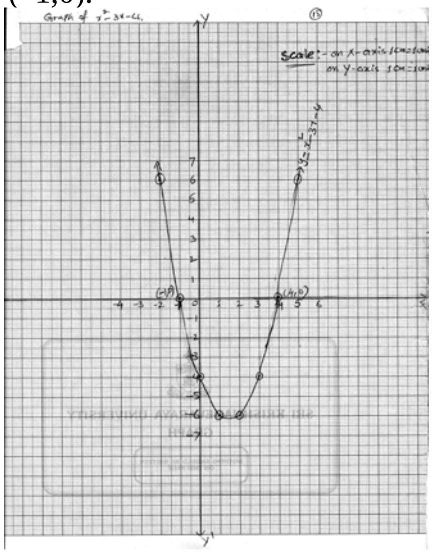
ANSWERS FOR 5 MARKS QUESTIONS (GRAPHICAL REPRESENTATION)

Draw the graphs of the quadratic polynomial and find the zeroes. Justify the answers.

i. $p(x) = x^2 - 3x - 4$

x	–2	–1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
–3x	6	3	0	–3	–6	–9	–12	–15
–4	–4	–4	–4	–4	–4	–4	–4	–4
$y = x^2 - 3x - 4$	6	0	–4	–6	–6	–4	0	6
(x,y)	(–2,6)	(–1,0)	(0,–4)	(1,–6)	(2,–6)	(3,–4)	(4,0)	(5,6)

The graph of p(x) (parabola) intersects the x-axis at (4,0) and (–1,0).



$\therefore$ The zeroes of $x^2 - 3x - 4$ are 4 and –1.

Verification :

Let $1x^2 - 3x - 4 = 0$ ($\because$ p(x) = 0)

$1x^2 - 4x + 1x - 4 = 0$

$x(x - 4) + 1(x - 4) = 0$

$(x - 4)(x + 1) = 0$

$x - 4 = 0$ (or) $x + 1 = 0$

$x = 4$ (or) $x = -1$

The zeroes of given quadratic polynomial are same by graphical representation as well as algebraic (factorization) method.

ii) $p(x) = y = x^2 - 6x + 9$

x	0	1	2	3	4	–1	–2	5
x^2	0	1	4	9	16	1	4	25
–6x	0	–6	–12	–18	–24	+6	+12	–30
+9	+9	+9	+9	+9	+9	+9	+9	+9
$y=x^2-6x+9$	9	4	1	0	1	16	25	4
(x,y)	(0,9)	(1,4)	(2,1)	(3,0)	(4,1)	(–1,16)	(–2,25)	(5,4)

Scale : on x-axis 1cm = 1 unit

on y-axis 1cm = 2 units

The graph of $y = x^2 - 6x + 9$ (parabola) intersects the x-axis at only one point (3,0)

$\therefore$ x - co-ordinate of the intersecting point is the zero of the given polynomial. i.e., 3 is only one zero of the p(x).

Justification :

Let $x^2 - 6x + 9 = 0$

$x^2 - 2.3.x + 3^2 = 0$

$(x-3)^2 = 0$

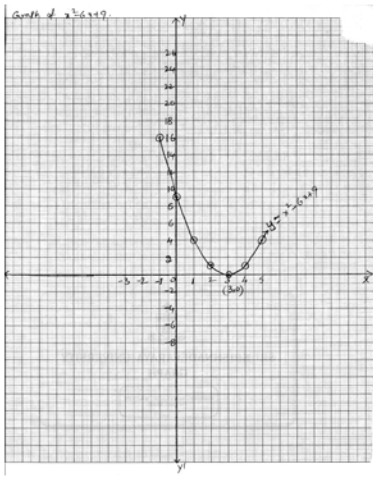
$x - 3 =$ ($a^2 - 2ab + b^2 = (a - b)^2$)

$x - 3 =$

$x = 3$

The zero of the given quadratic polynomial through the graph as well as algebraic method is same.

∴ Zero of the p(x) is true / correct.



iii. Given quadratic polynomial $p(x) = y = x^2 - x - 12$

x	–4	–3	–2	–1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
–x	4	3	2	1	0	–1	–2	–3	–4	–5
–12	–12	–12	–12	–12	–12	–12	–12	–12	–12	–12
$y=x^2-x-12$	8	0	–6	–10	–12	–12	–10	–6	0	8
(x,y)	(–4,8)	(–3,0)	(–2,–6)	(–1,–10)	(0,–12)	(1,–12)	(2,–10)	(3,–6)	(4,0)	(5,8)

Scale : on x-axis 1cm = 1 unit

on y-axis 1cm = 2 units

$\sqrt{0}$

The graph (parabola) of $p(x) = x^2 - x - 12$ intersects the x-axis at (–3,0) and (4,0) points

∴ The zeroes of $x^2 - x - 12$ are –3 and 4

$p(x) = x^2 - 1x - 12 = 0$

$x^2 - 4x + 3x - 12 = 0$

$x(x-4) + 3(x-4) = 0$

$(x-4)(x+3) = 0$

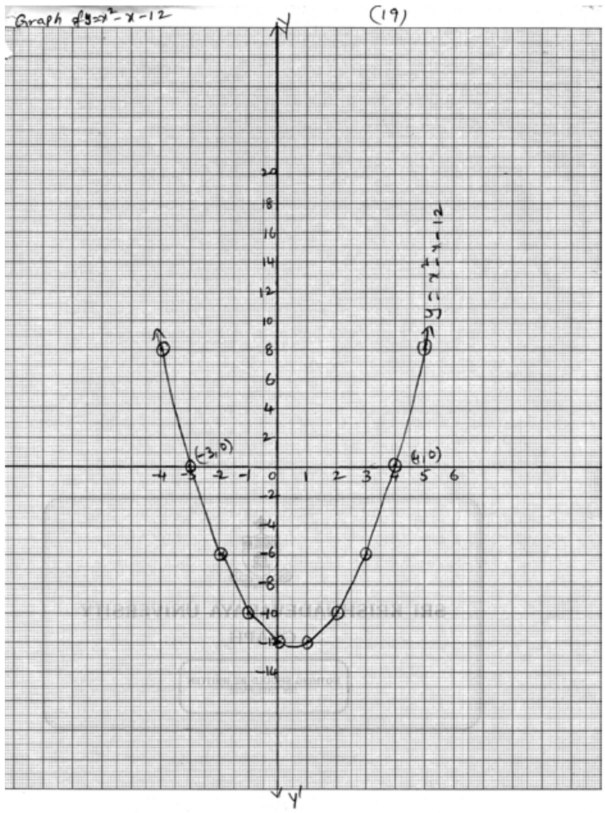
$x - 4 = 0$ (or) $x + 3 = 0$

$x = 4$ (or) $x = -3$

∴ Zeroes of p(x) are 4 and –3.

Finding the zeroes of p(x) through the graph and the method of factorization are same.

i.e., 4 and –3 are the zeroes of $x^2 - x - 12$.



CHAPTER - 4

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

The following type of questions asked for Exam :

2 mark questions – 1 – 2 x 1 = 2 Marks

4 mark questions – 2 – 4 x 2 = 8 Marks

5 mark questions – 1 – 5 x 1 = 5 Marks

$^1/_2$ mark questions – 3 – $^1/_2$ x 3 = $1^1/_2$ Marks

Total = $16^1/_2$ Marks

IMPORTANT POINTS :

- The general form of a Linear equation in two variables is ax + by + c = 0, a, b, c   R and $a^2 + b^2$ 0.
- The value of variables which satisfy both equations is called a solution of the pair of equations.
- Linear equations of two types :

  1) Consistent pair of linear equations

  2) Inconsistent pair of linear equations

1. Consistent pair of linear equations : A pair of equation which has atleast one solution. These are of two types :

   i) Mutually Independent Pair of Equations

   ii) Mutually Dependent Pair of Equations

- Mutually independent pair of linear equations has only one solution. If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are mutually independent then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
- Mutually dependent system of equations has Infinite solutions. If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are mutually dependent equations then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- Inconsistent pair of equations have no solutions. If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are inconsistent equations then .
- If a pair of inconsistent equations represents straight lines then they are parallel to each other.
- If consistent equations of mutually dependent represents straight lines, they coincide each other.
- If consistent equations of mutually dependent represents straight lines, they intersect at only one point.
- Pair of Linear of two variables can be solved by using graph, in substitution method or elimination method.

2 MARKS QUESTIONS (COMMUNICATION)

1. "In two supplementary angles one angle is 30^0 more than the second angle" write appropriate equations for the above.

Sol. Let the first angle = x^0

and second angle = y^0

Since x and y are supplementary angles,

$x + y = 180^0$ ....(1)

Since the first angle is 30^0 more than the second,

$x - y = 30^0$ ....(2)

$\therefore$ The linear equations are $x + y = 180$

$x - y = 30$

2. A shop keeper sold a chair and a table for Rs.570, hence he got 10% gain on chair and 15% gain on table. By gaining 15% on chair, 10% on table he sold the same for Rs.555. Write this information in the form of linear equations to find their C.P. (Comm.)

Sol. Let cost price of chair = Rs.x

cost price of table = Rs.y

S.P. of chair with 10% gain $= x \times \frac{100+10}{100}$

$= \frac{110x}{100}$ (or) $\frac{11x}{10}$

S.P. of table with 15% gain $= y \times \frac{100+15}{100}$

$= \frac{115y}{100}$ or $\frac{23y}{20}$

According to problem S.P. of both = Rs.570

i.e. $\frac{11x}{10} + \frac{23y}{20} = 570$ (or) $22x + 23y = 11400$ ....(1)

Similarly S.P. of both at 15% on chair and 10% on table is Rs.555

i.e., $\frac{23x}{20} + \frac{11y}{10} = 555$ (or) $23x + 22y = 11100$ ....(2)

$\therefore$ Linear equations are $22x + 23y = 11400$ and $23x + 22y = 11100$.

3. "The difference of two numbers is 26 and one number is 3 times of second number". Write the equations for above conditions with two variables x and y. (Comm.)

Sol. Let one number = x

Second number = y

Given that the difference of numbers is 26

i.e., $x - y = 26$ ....(1)

And first number is 3 times of second

i.e., $x = 3y$ (or) $x - 3y = 0$ ....(2)

The linear equations are $x - y = 26$ and $x - 3y = 0$.

4. 5 pencils and 7 pens together cost Rs.95. where as 7 pencils and 5 pens together cost Rs.85. Write the equations for finding the cost of each. (Comm.)

Sol. Let cost of a pencil = Rs.x

cost of a pen = Rs.y

By the problem cost of 5 pencils and 7 pens is 95.

i.e., 5.x + 7.y = 95 (or) 5x + 7y = 95 ....(1)

Cost of 7 pencils and 5 pens is Rs.85

i.e., 7. + 5.y = 85 (or) 7x + 5y = 85 ....(2)

$\therefore$ The equations are 5x + 7y = 95 and 7x + 5y = 85

5. Mary told her daughter, "seven years ago, I was seven times as old as you were then. Also three years from now, I shall be three times as old as you will be find the present age of Mary and her daughter, write the equations.

Sol. Let present age of Mary = x years

present age of daughter = y years

Seven years ago,

Mary's age = x – 7

Daughter's age = y – 7

Seven years ago Mary was 7 times as old as to her daughter.

i.e., x – 7 = 7(y – 7)

x – 7 = 7y – 49

x – 7y = –42 ....(1)

After 3 years,

Mary's age = x + 3

Daughter's age = y + 3

After 3 years now Mary will be 3 times as old as her daughter

i.e., x + 3 = 3(y + 3)

x + 3 = 3y + 9

x – 3y = 6 ....(2)

The linear equations are x – 7y = –42 and x – 3y = 6

4 MARKS QUESTIONS

1. Solve the following equations. (PS)

$$\frac{10}{x+y}+\frac{2}{x-y}=4 \;\; ; \;\; \frac{15}{x+y}-\frac{5}{x-y}=-2 .$$

Sol. Given are not Linear Equations, First change them into linear form.

$$\frac{10}{x+y}+\frac{2}{x-y}=4 \Rightarrow 10\left(\frac{1}{x+y}\right)+2\left(\frac{1}{x-y}\right)=4$$

$$\frac{15}{x+y}-\frac{5}{x-y}=-2 \Rightarrow 15\left(\frac{1}{x+y}\right)-5\left(\frac{1}{x-y}\right)=-2$$

Take $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$ and substitute, we get linear equations

10p + 2q = 4 ....(1)

15p – 5q = –2 ....(2)

(1)$\times$3 : 30p + 6q = 12

(2)   2 : 30p – 10q = –4

....

16q = 16

put q = 1 in (1) we get

10p + 2(1) =4

10p = 4 – 2

$q = \frac{1}{x-y} = 1 \Rightarrow x - y = 1$

$p = \frac{1}{x+y} = \frac{1}{5} \Rightarrow x + y = 5$

x + y = 5

x – y = 1

....

2x = 6

$x = \frac{6}{2} = 3$

put x = 3 in x + y = 5

3 + y = 5   y = 5 – 3 = 2

$\therefore$ The solution = (3,2)

2. Solve the equations $2^x + 3^y = 17$ and $2^{x+2} - 3^{y+1} = 5$. (PS)

Sol. $2^x + 3^y = 17$ and $2^{x+2} - 3^{y+1} = 5$

$2^x.2^2 - 3^y.3^1 = 5$

$4.2^x - 3.3^y = 5$

Let $2^x = p$ and $3^y = q$, we get linear equations

p + q = 17 ....(1) and

4p – 3q = 5 ....(2)

(1)   3 ....3p + 3q = 51

(2) 1 ....4p – 3q = 5

....

7p = 56

put p = 8 in (1)

8 + q = 17

q = 17 – 8 = 9

$\therefore$ p = 2^x = 8

$2^x = 2^3$ x = 3

q = 3^y = 9

$3^y = 3^2$ y = 2

$\therefore$ The solution = (3,2)

3. Solve the pair of equations 3x + 4y = 25 and 5x – 6y = –9 in substitution method. (PS)

Sol. 3x + 4y = 25 ....(1) and

5x – 6y = –9 ....(2)

Made either x or y as a subject from one equation and substitute in second

3x + 4y = 25

3x = 25 – 4y

$$\Rightarrow p = \frac{56}{7} = 8 \qquad x = \frac{25-4y}{3} \qquad y = \frac{-152}{-38}$$

Substitute x in (2)

$$5\left(\frac{25-4y}{3}\right) - 6y = -9$$

$$\frac{125-20y-18y}{3} = -9$$

125 – 38y = –27

–38y = –27–125

y = 4

3x + 4y = 25

3x + 4(4) = 25

3x = 25 – 16 = 9

$$x = \frac{9}{3} = 3$$

$\therefore$ The solution = (3, 4)

4. Solve and by elimination method. (PS)

Sol. ....(1)

........(2)

(1) 3 ....

(2) 1.........

....

put y = 2 in (1)

26 68826
3x + y = 13 658 25
y 2y 13

x = 6 – 3 = 3

∴ The solution = (3,2)

5. The sum of two digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number.

Sol. Let the digit in units place = x

digit in tens place = y

Then the two digit number = 10(y) + 1(x)

= x + 10y

By reversing the digits, the so formed number = 10(x) + 1(y)

= 10x + y

Given that the sum of both numbers is 66

i.e., x + 10y + 10x + y = 66

11x + 11y = 66 (or)

x + y = 6 ....(1)

But difference of ditis is 2

i.e., x – y = 2 ....(2)

x + y = 6

x – y = 2

....

2x = 8

put x = 4 in (1)

x + y = 6

4 + y = 6

y = 6 – 4 = 2

∴ The two digit number is 24.

REASONING AND PROOFS

1. For what positive value of 'p' the following pair of linear equations have infinitely many solutions, and verify it. px + 3y – (p – 3) = 0 and 12x + py – p = 0. (RP)

Sol. px + 3y – (p – 3) = 0 ....(1)

12x + py – p = 0 ....(2)

$a_1 = p$; $b_1 = 3$; $c_1 = -(p-3) = 3 - p$

$a_2 = 12$; $b_2 = p$; $c_2 = -p$

Since the pair of equations have infinite solutions, the relation between the coefficients is

$= \frac{3}{6} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$

∴ positive value of p = 6

Check : substituting p = 6 in the equations we get

6x + 3y – (6 – 3) = 0

6x + 3y – 3 = 0 ....(1)

12x + 6y – 6 = 0 ....(2)

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

2. Verify the following equations are consistent or inconsistent. If consistent solve them. (RP)

$2x + y = 5$ and $3x - 2y = 4$

Sol. Writing the equations in standard form.

$2x + y - 5 = 0$ ....(1)

$3x - 2y - 4 = 0$ .....(2)

$a_1 = 2$; $b_1 = 1$; $c_1 = -5$

$a_2 = 3$; $b_2 = -2$; $c_2 = -4$

Here

∴ Given pair of equations are consistent.

It has only one solution

(1) $\times$ 2.........$4x + 2y - 10 = 0$

(2) ....$3x - 2y - 4 = 0$

....

$7x \quad - 14 = 0$

$7x = 14$

$$x = \frac{14}{7} = 2$$

$$\frac{a_1}{a_2} = \frac{2}{3} ; \frac{b_1}{b_2} = \frac{1}{-2} \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

put $x = 2$ in (1)

$2(2) + y - 5 = 0$

$4 + y - 5 = 0$

$y - 1 = 0$

$y = 1$

∴ The solution = (2, 1)

3. Five years ago A's age was three times of B. Ten years later A will be two times of B's. Find present ages of A and B. (Conn)

Sol. Let present age of A = x years

present age of B = y years

5 years ago,

A's age = $x - 5$

B's age = $y - 5$

By the problem A was 3 times of B

i.e., $x - 5 = 3(y - 5)$

$x - 3y = -10$ .... (1)

After 10 years,

A's age = x + 10
B's age = y + 10
A's age is 2 times of B
i.e., x + 10 = 2(y + 10)
x – 2y = 20 – 10
x – 2y = 10 .... (2)
x – 3y = –10
x + 2y = 10
....
–y = –20
y = 20
put y = 20 in (1)
x – 3(20) = –10
x – 60 = –10
x = –10 + 60 = 50
∴ present age of A = 50 years
present age of B = 20 years.

4. A fraction becomes if 1 is added to both numerator and denominator. If, however 5 is subtracted from both numerator and denominator, the fraction becomes $\frac{4}{5}$ $\frac{1}{2}$ $\frac{x}{y}$ i.e., $\frac{x+1}{y+1}=\frac{4}{5}$. What is the fraction. (Conn.)

Sol. Let the numerator = x
and denominator = y

Then the fraction

If 1 is added to both N & D, the fraction is

$\Rightarrow$ 5x + 5 = 4y + 4
5x – 4y = –1 ....(1)

If 5 is subtracted from N & D, the fraction is

$\Rightarrow$ 2x – 10 = y – 5

2x – y = 5 ....(2)

(1) 1 : 5x – 4y = –1

(2) 4 : 8x – 4y = 20

....

–3x = –21

put x = 7 in (1) we get

5(7) – 4y = –1

35 – 4y = –1

–4y = –36

$$y = \frac{-36}{-4} = 9$$

$\therefore$ The fraction is $\frac{7}{9}$.

5. Neha went to a 'sale' to purchase some pants and skirts. When her friend asked her how many of each she had bought, she answered "The number of skirts are two less than twice the number of pants purchased. Also the number of skirts is four less than four times the number of pants purchased." Help her friend to find how many pants and skirts Neha bought. (Conn)

Sol. Let number of pants = x

$\Rightarrow x = \frac{-21}{-3} = 7$

$x = \frac{-2}{-2} = 1$

and number of skirts = y

By the problem skirts are equal to two less than two times of pants.

i.e., y = 2x – 2

2x – y = 2 ....(1)

And skirts equal to four less than four times of pants

i.e., y = 4x – 4

4x – y = 4 ....(2)

2x – y = 2

4x – y = 4

....

–2x = –2

put x = 1 in (1)

2(1) – y = 2

–y = 2 – 2 = 0

y = 0

$\therefore$ Number of pants = 1

skirts = 0.

6. Compare the coefficients and fill the blank in the table. (RP).

Pair of line	$\frac{a_1}{a_2}$			comparison of ratios	graphical representation	algebraic interpretation
1) $2x + y - 5 = 0$ $3x - 2y - 4 = 0$		(1)			(2)	unique solution
2) $3x + 4y - 2 = 0$ $6x + 8y - 4 = 0$	(3)	$\frac{4}{8}$		(4)	(5)	(6)
3) $4x - 6y - 15 = 0$ $2x - 3y - 5 = 0$		(7)	(8)		(9)	Infinite solution

7. We have a linear equation $2x + 3y - 8 = 0$. Write another linear equation in two variables such that the pair of equations form. (a) consistent pair, (b) dependent pair. (RP)

Sol. Given linear equation is

$2x + 3y - 8 = 0$, comparing with $a_1x + b_1y + c_1 = 0$

we get $a_1 = 2$, $b_1 = 3$, $c_1 = -8$ $\quad$ i.e., $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, $\frac{2}{a_2} = \frac{3}{b_2} = \frac{-8}{c_2}$

a) The relation between the coefficients if the equations are consistent is

i.e., $\frac{2}{a_2} \neq \frac{3}{b_2} \Rightarrow \frac{a_2}{b_2} \neq \frac{2}{3}$

Take

Then $a_2 = 4$ and $b_2 = 5$ and c_2 = any real no.

$\therefore$ The required second equation is $4x + 5y + 7 = 0$

b) The relation between the coefficients if the equations are dependent pair.

$\frac{2}{a_2} = \frac{3}{b_2} \Rightarrow \frac{a_2}{b_2} = \frac{2}{3}$

Take then

$\therefore$ The required linear equation is

$t = a_2x + b_2y + c_2 = 0$

i.e., $4x + 6y - 16 = 0$.

5 MARKS QUESTIONS (REPRESENTATION)

1. Solve by graphical method : $2x + 3y = 1$ and $3x - y = 7$.

Sol. $2x + 3y = 1$ ....(1)

$3x - y = 7$ ....(2)

$2x + 3y = 1$ ....(1)

$3y = 1 - 2x$

x	2	–1	–4
y	–1	1	3

Graph $y = \frac{1-2x}{3}$ $\frac{2}{4} = \frac{3}{6} = \frac{-8}{c_2} \Rightarrow c_2 = -16$

$3x - y = 7$ ....(2)

$y = 3x - 7$

x	1	2	3
y	–4	–1	2

solution = (2, –1)

2. Solve the following equations using graph. (Rep)

$2x + y - 6 = 0$ and $4x - 2y - 4 = 0$

Sol. $2x + y - 6 = 0$ ....(1)

$4x - 2y - 4 = 0$ ....(2)

$2x + y - 6 = 0$

$y = 6 - 2x$

x	1	2	4
y	4	2	–2

$4x - 2y - 4 = 0$

$-2y = 4 - 4x$

$$y = \frac{4-4x}{-2}$$

$= \frac{4x-4}{2}$

Graph

x	0	1	2
y	–2	0	2

$\therefore$ Solution = (2, 2)

3. The area of a rectangle gets reduced by 80 sq.units if its length is reduced by 5 units and breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, the area will increase by 50 sq.units. Find the length and breadth using Graph. (Rep)

Sol. Let the length = x units

breadth = y units

Then area = xy sq.units

If length decreased 5 units, breadth increased 2 units then area 80 sq.u. less than original.

i.e., $(x - 5)(y + 2) = xy - 80$

$xy + 2x - 5y - 10 = xy - 80$

$xy + 2x - 5y - xy = -80+10$

$2x - 5y = -70$ ....(1)

If length increased 10 units, breadth decrease 5 units then area increases by 50 sq.units

i.e., $(x + 10)(y - 5) = xy + 50$

$xy - 5x + 10y - 50 - xy = 50$

$-5x + 10y = 50 + 50$

$-5x + 10y = 100$

$-x + 2y = 20$ ....(2)

$2x - 5y = -70$ ....(1)

$5y = 2x + 70$

$y = \frac{2x+70}{5}$

x	–10	–5	0
y	10	12	14

Graph

solution = (40, 30)

$\therefore$ Length = 40 units

breadth = 30 units

4. Solve the following by using graph. (Rep)

   $4x - y = 16$ and $\frac{3x-7}{2} = y$

5. By using graph solve the following. (Rep)

   $5x + 2y = 1$ and $7x + 3y = -1$

6. Tabita went to a bank to withdraw Rs.2000. She asked the cashier to give the cash in Rs.50 and Rs.100 notes only. Snigdha got 25 notes in all can tell howmany notes each of Rs.50 and Rs.100. She received ? Solve by using graph.

OBJECTIVE QUESTIONS

1. If $3x + 2ky - 2 = 0$ and $2x + 5y + 1 = 0$ are two parallel lines then the value of k is ....

   A) 2 B) C) 1 D) ( )

2. If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ represents the same line then .... ( )

   A) B) C) D)

3. The equation which makes a consistent pair with $2x + y - 5 = 0$ in the following is ..... ( )

   A) $3x - 2y - 4 = 0$ B) $4x + 2y - 10 = 0$

   $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

   C) $20x + 10y - 50 = 0$ D) $3x + 4y - 20 = 0$

4. In the following the mutually dependent pair is .... ( )

   A) $2x + 3y - 5 = 0, 3x - 4y - 5 = 0$ B) $3x - 2y + 4 = 0, 6x - 4y + 8 = 0$

   C) $x + y - 3 = 0, 5x - 3y + 2 = 0$ D) $3x - 2y - 5 = 0, 4x + 3y + 2 = 0$

5. In the given below which is not a linear equation ? ( )

   A) $x^2 = 2y + 3$ B) $3y - 4 = x$ C) $4x + 3 = y - 1$ D) $x^3 = 1 + y$

6. In the adjoining diagram the point where the line cuts the x-axis is .... ( )

   A) (0, 3)

   B) (3, 0)

   C) (0, 0)

   D) (3, 3)

7. The equation that cuts the y-axis at (0, 5) is .... ( )

   A) $x + 5 = 0$ B) $y - 5 = 0$ C) $x = 0$ D) $y + 3 = x$

8. The graphic representing the equations $x + 3y = 6$ and $4x + 12y = 8$ are .... ( )

   A) // lines B) intersecting lines C) coinciding lines D) None

9. The consistent pair of equations are .... ( )

A) parallel lines B) intersecting lines C) coinciding lines D) None

10. The quadrant that lie (2, –3) is .... ( )

A) I B) II C) III D) IV

11. If x + 3y = 4 and 5x + py = 20 represents a pair of inconsistent system then the value of p is ....

12. A pair of equations 2x + ky – 1 = 0 and 5x + 7y + 7 = 0 has only one solution then k ....

13. If px + qx + r = 0 and ax + by + c = 0 are parallel lines then relation between coefficients is ....

14. The pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has no solution then $a_1 : a_2 =$ ....

15. If $L_1 = a_1x + b_1y + c_1 = 0$ and $L_2 = a_2x + b_2y + c_2 = 0$ and $L_1 // L_2$ then .... (relation between coefficients)

OBJECTIVE QUESTIONS ANSWERS

1) C	2) A	3) A	4) V	5) A
6) B	7) B	8) A	9) B	10) D
11) 15	12)	13)	14) $b_1 : b_2$	
15)				

$$\neq \frac{14}{5}$$

$$\frac{p}{a} = \frac{q}{b} \neq \frac{v}{c}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

CHAPTER - 5

QUADRATIC EQUATIONS

1. **Marks Weightage :**

   No. of questions asked for 2 marks = 1

   No. of questions asked for 4 marks = 1

   No. of bits asked ($^1/_2$ mark) = 3 to 4

   Total weightage marks from this chapter = $7^1/_2$ to 8 marks

2. This chapter may be covered under Group - A of Section - I and Section - III for Part - A.

3. **Concepts and Formulaes :**

   i) Any equation of the form p(x) = 0, where p(x) is polynomial of degree 2, is a quadratic equation.

   ii) When we write the terms of p(x) in descending order of their degrees, then we get the standard form of the equation.

   i.e., $p(x) = ax^2 + bx + c = 0$, $(a \neq 0)$ is called the standard form of a quadratic equation, but $p(x) = y = ax^2 + bx + c$ $(a \neq 0)$ is called a quadratic function.

   iii) The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of quadratic equation $ax^2 + bx + c = 0$ are the same.

   iv) Methods of solving the quadratic equations :

   a) Factorization Method : We have found the roots of $ax^2 + bx + c = 0$ by factorising $ax^2 + bx + c$ into product of two linear factors and equating each factor to zero.

   b) The method of completing the square (by using identities such as $(a+b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$) can be used for solving quadratic equation.

   c) Quadratic Formula : The roots of quadratic equation $ax^2 + bx + c = 0$ are given by

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, provided $b^2 - 4ac \geq 0$.

   v) Since $b^2 - 4ac$ determines whether the quadratic equation $ax^2 + bx + c = 0$ has real roots or not, $b^2 - 4ac$ is called 'Discriminant' of the quadratic equation.

   vi) A quadratic equation $ax^2 + bx + c = 0$ has

   a) Two distinct real roots, if $b^2 - 4axc > 0$ (positive value)

   b) Two equal (coincident) roots, if $b^2 - 4ac = 0$ and

   c) No real roots (complex numbers), if $b^2 - 4ac < 0$ (negative value).

2 MARKS QUESTIONS

1. Find the roots of the quadratic equation : $x^2 - 3x - 10 = 0$. (PS)
2. Find two numbers whose sum is 27 and product is 182. (PS)
3. If 2 and 3 are the roots of quadratic equation $3x^2 - 2kx + 2m = 0$, find the values of k and m ? (PS)
4. Find the roots of $4x^2 + 3x + 5 = 0$ by the method of completing the square. (PS)
5. Find the roots of the equation : $x + \frac{1}{x} = 3 \ (x \neq 0)$. (PS)
6. Find the nature of the roots of the quadratic equations i) $3x^2 - 4\sqrt{3}x + 4 = 0$, ii) $2x^2 - 6x + 3 = 0$.

7. Find the values of K for the quadratic equation : $2x^2 + Kx + 3 = 0$, so that it has two equal roots.

8. Find the discriminant of the equation $3x^2 - 2x +$ $= 0$ and hence find the nature of its roots. Find them, if they are real. (PS & Comm)

4 MARKS QUESTIONS

9. Sum of the areas of two squares is $468 m^2$. If the difference of their perimeters is 24m, find the sides of the two squares. (PS & Comm)

10. If a polygon of 'n' sides has $\frac{1}{2}n(n-3)$ diagonals. How many sides will a polygon having 65 diagonals ? Is there a polygon with 50 diagonals ? (R.P. & Conn)

11. The base of a triangle is 4cm longer than its altitude. If the area of the triangle is 48 sq.cm, then find its base and altitude ? (R.P. & Comm)

12. A motor boat whose speed is 18km/h in still water. It takes 1 hour more to go 24km upstream than to return down stream to the same spot. Find the speed of the stream. (R.P. & Conn.)

13. Find the roots of the equation : (PS).

14. If $x - \frac{3}{x} = 2$, find x values. (PS)

15. Whether $(x+1)^2 = 2(x-3)$ is quadratic equation or not ? Verify.

16. Solve : . (For 4 marks) (PS)

[illegible]

BITS

I. Multiple Choice :

1. Which of the following is not a quadratic equation ? ( )

   A) $x(2x+3) = x^2 + 1$ B) $(x-2)^2 + 1 = 2x - 3$

   C) $x^2 + 3x + 1 = (x-2)^2$ D) $(x+1)^2 = 2(x-3)$

2. Which of the following is a quadratic equation? ( )

   A) $(x^2+1)(x^2-1) = 0$ B) $(x-2)^3 = 8$

   C) $x(x+1) + 8 = (x+1)(x-2)$ D) $x^2 - 55x + 750 = 0$

3. The sum of a number and its reciprocal is $\frac{10}{3}$. The quadratic equation that represents the situation is ...... ( )

   A) B) C) D)

4. If α, β are the roots of $x^2 + 7x - 60 = 0$, then the value of $\alpha + \beta + \alpha\beta =$ ...... ( )

   A) –67 B) –53 C) 53 D) 67

5. The quadratic equation $px^2 + qx + r = 0$ has two distinct real roots, if .... ( )

   A) $q^2 = 4pr$ B) $q = 2pr$ C) $q^2 > 4pr$ D) $q^2 < 4pr$

II. Fill in the blanks :

6. If one root of $x^2 + px + 3 = 0$ is 1, then the values of 'p' is ....
7. The quadratic equation with roots α and β is ....
8. The standard form of a quadratic equation in y is ....
9. The condition for a quadratic equation to have imaginary (complex) roots is ....
10. Product of the roots of $3x^2 - 5x + 2 = 0$ is ....
11. Sum of the roots of $x^2 + 3x - 10 = 0$ ....

III. Matching :

If $D = b^2 - 4ac$ is discriminant of $ax^2 + bx + c = 0$,

	Group - A		Group - B
12.	If $D > 0$	[ ]	A) The curve of the quadratic equation touches x-axis at one point
13.	If $D = 0$	[ ]	B) The curve of the quadratic polynomial does not touch x-axis at all
14.	If $D < 0$	[ ]	C) The curve of quadratic polynomial cuts the x-axis at two points.
15.	D of $2x^2 + 3x + 1 = 0$ is ...	[ ]	D) 0
16.	D of $x^2 - \sqrt{2}x + \frac{1}{2} = 0$ is	[ ]	E) 1
			F) $=\sqrt{2}$

ANSWERS (2 MARKS)

1. Given quadratic equation : $1x^2 - 3x - 10 = 0$

   $1x^2 - 5x + 2x - 10 = 0$

   $x(x - 5) + 2(x - 5) = 0$

   $(x - 5)(x + 2) = 0$

   $x - 5 = 0$ (or) $x + 2 = 0$

   $x = 5$ (or) $x = -2$

   $\therefore$ –2 and 5 are roots of quadratic equation.

2. Say two numbers as x, y

   Their sum : $x + y = 27$ (given)

   $y = 27 - x$ .... (1)

   Their product : $xy = 182$ (given) ....(2)

   substitute $y = 27 - x$ in eq. (2)

   $x(27 - x) = 182$

   $27 - x^2 = 182$

   $1x^2 - 27x + 182 = 0$

   $x^2 - 13x - 14x + 182 = 0$

$x(x - 13) - 14(x - 13) = 0$

$(x - 13)(x - 14) = 0$

$x - 13 = 0$ (or) $x - 14 = 0$

$x = 13 \qquad x = 14.$

$\therefore$ Required two numbers are $x = 13, y = 14$ (or) $x = 14, y = 13$.

3. If 2 is one root of quadratic equation : $p(x) = 3x^2 - 2kx + 2m = 0$, then $p(2) = 0$

$3(2)^2 - 2k(2) + 2m = 0$

$12 - 4k + 2m = 0$

$-4k + 2m = -12$ ......(1)

similarly $p(3) = 0$

$3(3)^2 - 2k(3) + 2m = 0$

$27 - 6k + 2m = 0$

$-6k + 2m = -27$ ....(2)

subtract eq. (1) from eq.(2)

$-6k + 2m = -27$

$-4k + 2m = -12$

....

$-2k \qquad = -15$

$\Rightarrow k = \frac{15}{2}$

From eq.(1), $-4\left(\frac{15}{2}\right) + 2m = -12$

$-30m + 2m = -12$

$2m = -12 + 30$

$2m = 18$

$m = \frac{18}{2} = 9$

$\therefore k = \frac{15}{2}, m = 9$

4. Given quadratic equation : $4x^2 + 3x + 5 = 0$

Dividing by 4 on both sides

$x^2 + \frac{3}{4}x + \frac{5}{4} = 0$

$$x^2 + 2.x.\frac{3}{4.2} = \frac{-5}{4}$$

Add $\left(\frac{3}{8}\right)^2$ on both sides

$$x^2 + 2.x.\frac{3}{8} + \left(\frac{3}{8}\right)^2 = \frac{-80+9}{64}$$

$$\left(x + \frac{3}{8}\right)^2 = \frac{-71}{64} < 0$$

So, there is no real value of x.

satisfying the given equation,

Therefore, the given equation has no real roots.

5. Given quadratic equation : $x + \frac{1}{x} = 3 \quad (x \neq 0)$

$$\frac{x^2+1}{x} = 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + 2.x.\frac{3}{8} + \left(\frac{3}{8}\right)^2 = \frac{-5}{4} + \frac{9}{64}$$

$x^2 + 1 = 3x$

$1x^2 - 3x + 1 = 0$

comparing above equation with $ax^2 + bx + c = 0$

we get a = 1, b = –3, c = 1

using quadratic formula :

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

So, the roots are $\frac{3+\sqrt{5}}{2}$ and $\frac{3-\sqrt{5}}{2}$.

6.i. Given quadratic equation : $3x^2 - 4\sqrt{3}x + 4 = 0$

comparing it with $ax^2 + bx + c = 0$

we get a = 3, , c = 4

So, its discriminant = $b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4)$

= 48 – 48

= 0.

Since discriminant = 0 ; the given equation has two equal real roots.

ii. Given quadratic equation : $2x^2 - 6x + 3 = 0$

comparing it, with $ax^2 + bx + c = 0$

we get a = 2, b = –6, c = 3

So, its discriminant = $b^2 - 4ac = (-6)^2 - 4(2)(3)$

= 36 – 24

= 12 > 0

∴ The given equation has two distinct real roots.

7. Given quadratic equation : $2x^2 + kx + 3 = 0$

comparing it, with $ax^2 + bx + c = 0$,

we get a = 2, b = k, c = 3

Given that it has two equal roots, $K = \pm 2\sqrt{6}$ $c = \left(\frac{1}{3}\right) \frac{1}{3}$

So, its discriminant = $b^2 - 4ac = 0$

$K^2 - 4(2)(3) = 0$

$K^2 - 24 = 0$

$K^2 = 24$

$K = \sqrt{24} = \sqrt{4 \times 6}$

When $K = 2\sqrt{6}$ (or) $-2\sqrt{6}$, the quadratic equation has two equal roots.

8. Given quadratic equation : $3x^2 - 2x + \frac{1}{3} = 0$

comparing it, with $ax^2 + bx + c = 0$,

we get, a = 3, b = –2,

Its discriminant : $b^2 - 4ac = (-2)^2 - 4(3)$

= 4 – 4

= 0

$\therefore$ The roots of the quadratic equation has two equal real numbers

$\therefore$ Quadratic Formula :

$$= \frac{-(-2) \pm 0}{2(3)}$$

$$= \frac{2}{2(3)} = \frac{1}{3}$$

is only one root of the quadratic equation.

ANSWERS (4 MARKS)

9. Let the length of the side of smaller square = x meters
Then its perimeter = 4.x = 4x meters
The perimeter of the larger square = (4x + 24) meters

$\therefore$ The length of the side of larger square

= (x + 6) meters
Area of the smaller square = x^2
Area of the larger square = $(x + 6)^2 = x^2 + 12x + 36$ $\Rightarrow \left(\frac{4x+24}{4}\right)$ $x = \frac{1}{3}$ $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
= $2x^2 + 12x + 36$
By problem,
$2x^2 + 12x + 36 = 468$
$2x^2 + 12x + 36 - 468 = 0$
$2x^2 + 12x - 432 = 0$
$1x^2 + 6x - 216 = 0$
(Dividing by 2 on both sides)
comparing the above, equation with $ax^2 + bx + c = 0$,
we get a = 1, b = 6, c = –216
using the quadratic formula, we get

$$= \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-216)}}{2.1}$$

$$= \frac{-6 \pm \sqrt{36 + 864}}{2}$$

$$= \frac{-6 \pm \sqrt{900}}{2}$$

$= \frac{-6 \pm 30}{2}$

$= -3 \pm 15$

$\therefore$ x = –3 + 15 (or) –3 –15

= 12 (or) –18 is ignored

(since sides of a square can't be negative)

we take x = 12 meters

The length of the side of a smaller square = 12 mts

The length of the side of a larger square = (x + 6)

= 12 + 6

= 18 mts.

10. The number of diagonals of a polygon with 'n' sides

Given that no. of diagonals of a polygon = 65

$n^2 - 3n = 130$

$1n^2 - 3n - 130 = 0$

$1n^2 - 13n + 10n - 130 = 0$

$n(n - 13) + 10(n - 13) = 0$

$(n - 13)(n + 10) = 0$

$n - 13 = 0$ (or) $n + 10 = 0$

$n = 13$ (or) $n = -10$ is ignored. (number of sides is never negatives)

$\frac{n(n-3)}{2}$

$\therefore \frac{n(n-3)}{2} = 65$

$\frac{n(n-3)}{2} = 50$

$\Rightarrow n^2 = 3n + 100$

$n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-100)}}{2(1)}$

So, we take n = 13

Hence, the number of sides of the required polygon is 13.

There is no polygon with 50 diagonals.

Explanation :

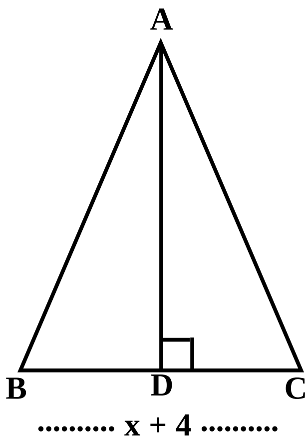
$\Rightarrow 1n^2 - 3n - 100 = 0$

$= \frac{3 \pm \sqrt{9 + 400}}{2}$

$n = \frac{3 \pm \sqrt{409}}{2}$

$\therefore$ 'n' has no real value as integer.

11. In ΔABC, let AD be altitude and BC be the base.



The base of a triangle is 4cm longer than its altitude.

If AD = x, then BC = x + 4

Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{altitude}$

$= \frac{1}{2} \times (x+4)x \quad \left(= \frac{1}{2} \times BC \times AD \right)$

By problem

x(x+4) = 96

$1x^2 + 4x - 96 = 0$

$1x^2 + 12x - 8x - 96 = 0$

x(x + 12) – 8(x + 12) = 0

(x + 12) (x – 8) = 0

x + 12 = 0 (or) x – 8 = 0

x = –12 (or) x = 8

(ignored)

(The length of altitude can't be negative)

$= \frac{1}{2}(x+4)x = 48$

$\Rightarrow \frac{\text{distance}}{\text{speed}} = \frac{24}{(18-x)}$ hours

Its altitude (x) = 8cm

Base (x+4) = 8 + 4 = 12cm

12. Let the speed of the stream be x km/h

The speed of the boat upstream = (18 – x) km/h and

The speed of the boat down stream = (18 + x) km/h

The time taken to go upstream

The time taken to go down stream $= \frac{24}{(18-x)}$ hours

According to the question,

$\frac{24}{18-x} - \frac{24}{18+x} = 1$

24 (18 + x) – 24 (18 – x) = (18 – x) (18 + x)

$432 + 24x - 432 + 24x = 324 - x^2$

$1x^2 + 48x - 324 = 0$

comparing this with $ax^2 + bx + c = 0$, we get

a = 1, b = 48, c = –324

a. Formula : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-48 \pm \sqrt{(48)^2 - 4(1)(-324)}}{2(1)}$

$= \frac{-48 \pm \sqrt{2304 + 1296}}{2}$

$= \frac{-48 \pm \sqrt{3600}}{2}$

$= \frac{-48 \pm 60}{2}$

$= \frac{-48 + 60}{2}$ (or) $\frac{-48 - 60}{2}$

$\Rightarrow \frac{12}{2}$ (or) $\frac{-108}{2}$

$\Rightarrow 6$ (or) -54

Since x is the speed of the stream, it cannot be negative.

So, we ignore the root $x = -54$

$\therefore$ x = 6 gives the speed of the stream as 6km/h.

$x - \frac{3}{x} = 2 \Rightarrow \frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30} (x \neq -4, 7)$

13. Given equation :

$\frac{x - 7 - x - 4}{(x+4)(x-7)} = \frac{11}{30}$

$-11 \times 30 = 11(x + 4)(x - 7)$

$-30 = (x + 4)(x - 7)$

$x^2 - 7x + 4x - 28 + 30 = 0$

$1x^2 - 3x + 2 = 0$

$1x^2 - 2x - 1x + 2 = 0$

$x(x - 2) - 1(x - 2) = 0$

$(x - 2)(x - 1) = 0$

$\therefore x - 2 = 0$ (or) $x - 1 = 0$

$x = 2$ (or) $x = 1$

$\therefore$ 1 and 2 are the roots of the given quadratic equation.

14. $x^2 - 3 = 2x$ (multiplying by x on both sides)

$1x^2 - 2x - 3 = 0$

$x^2 - 3x + 1x - 3 = 0$

$x(x - 3) + 1(x - 3) = 0$

$(x - 3)(x + 1) = 0$

$x - 3 = 0$ (or) $x + 1 = 0$

$x = 3$ (or) $x = -1$

15. $(x + 1)^2 = 2(x - 3)$ $\quad x^2 + 2x + 1 = 2x - 6$

$x^2 + 2x + 1 - 2x + 6 = 0$

$x^2 + 7 = 0 \quad x^2 + 0.x + 7 = 0$

It is in the form of $ax^2 + bx + c = 0$

$\therefore$ The given equation is a quadratic equation.

16.

$$\frac{(x-1)(x-4)+(x+2)(x-3)}{(x+2)(x-4)} = \frac{10}{3}$$

$$\frac{x^2-5x+4+x^2-1x-6}{x^2-2x-8} = \frac{10}{3}$$

$$\frac{\not{x}-1}{x+2} + \frac{x-3}{x-4} = \frac{10}{3}$$

$3(2x^2 - 6x - 2) = 10(x^2 - 2x - 8)$

$6x^2 - 18x - 6 = 10x^2 - 20x - 80$

$10x^2 - 6x^2 - 20 + 18x - 80 + 6 = 0$

$4x^2 - 2x - 74 = 0$

$2x^2 - x - 37 = 0$

Comparing the above equation with $ax^2 + bx + c = 0$

we get $a = -2, b = -1, c = -37$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-37)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{497}}{4}$$

ANSWERS : BITS

I. 1) C 2) D 3) C 4) A 5) C

II. 7. $x^2 - x(\alpha + \beta) + \alpha.\beta = 0$

6. –4 8) $ay^2 + by + c = 0$ 9) Discriminant = $b^2 - 4ac > 0$

10. $\frac{2}{3}$ 11) –3

III. 12) C 13) A 14) B 15) E 16) D.

Note : Model of question patterns are supplied butnot given as it is questions in the public examinations.

CHAPTER - 6
PROGRESSIONS

From this Chapter, there is a possibility of getting 9 marks from Part A & Part B.

2 marks questions – problem solving

1 mark questions – problem solving

4 marks question –

Part B - 3 or 4 bits.

Important Concepts :

Arithmetic Progression (A.P.) :

1. An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixing number d to the preceeding term, except the first term. The fixed number d is called the common difference.

   The terms of AP are a, a+d, a+2d, a+3d.......

2. nth term of AP (general form)

   $a_n = a + (n-1)d$

3. The sum of the first n terms of an AP is given by

   $$S_n = \frac{n}{2}[2a + (n-1)d]$$

4. If l is the last term of the finite AP, say the nth term, then the sum of all terms of the AP is given by

   $$S = \frac{n}{2}[a + l]$$

Geometric Progression (G.P.)

1. A Geometric Progression (G.P.) is a list of numbers in which each term is obtained by multiplying preceeding term with a fixed number 'r' except first term. This fixed number is called common ratio 'r'.

   The general form of G.P. is a, ar, ar^2....

2. In the first term and common ratio of a G.P. are a, r respectively then the nth term.

   $a_n = a.r^{n-1}$

Exercise questions on key concepts :

Sl.	Formula	Application
1.	In an A.P. $a_n = a + (n-1)d$	i) 18th term of 16, 11, 6, 1 .... (–69) ii) nth term of 16, 11, 6, 1 .... (21-5n)
2.	In an –A.P.	i) Sum of multiples of 3 between 1 and 100 .... (1683) ii) The sum of the nutural numbers from 1 to 100 .... (5050)

3.	Three terms of – A.P.	3) The sum of 3 terms in A.P. is 21 and their product is 315 then those terms .... (5, 7, 9)
4.	In G.P. $a_n = ar^{n-1}$	4) In a G.P., nth term in 2 $(0.5)^{n-1}$ then its first term .... and common ratio ....
5.	If a1, a2, a3 are the consecutive numbers then $a_2^2 = a_1.a_3$	5) If – 2/7, x, –7/2 are the consecutive numbers of a G.P. then x .... ($\pm$1)
6.	If an A.P., 4th terms 7 and 7th terms 4, ten show that its 11th term is zero	b) an = a + (n–1)d a4 = a + 3d = 7 –3d = 3 $\therefore$ d = 3/–3 = –1 Substituting of in (1) a + 3 (–1) = 7 a – 3 = 7 $\Rightarrow$ a = 7 + 3 = 10 a_{11} = a + 10 d = 10 + 10 (–1) = 10 – 10 = D

TWO MARKS QUESTIONS

1. How many two digit numbers are divisible by 3 ? (P.S.)

A. List of two digit numbers that are divisible by 3 are

12, 15, 18,..........99

The above list of the numbers is in A.P.

So that first term (a) = 12

common difference (d) = 3

$= \frac{87}{3} = 29$ $\quad \frac{P \times R \times T}{100}$

last term = a_n = 99

In an A.P., a_n = a + (n – 1)d

99 = 12 + (n – 1)x3

(n – 1)x3 = 99 – 12 = 87

(n – 1)

$\therefore$ n = 29 + 1 = 30.

2. A sum of Rs. 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an A.P. ? If so, find the interest at the end of 30 years ? (Connection)

A. We know that the formula to calculate simple interest is given by

simple interest

So, the interest at the end of the 1st year $= \text{Rs.}\frac{1000 \times 8 \times 1}{100} = \text{Rs.}80$

The interest at the end of the 2nd year $= \text{Rs.}\frac{1000 \times 8 \times 2}{100} = \text{Rs.}160$

The interest at the end of the 3rd year v

Similarly, we can obtain the interest at the end of the 4th year, 5th year and so on. So the interest (in Rs.) at the end of the 1st, 2nd, 3rd,.... years respectively, are 80, 160, 240.

It is an A.P. as the difference between the consecutive terms in the list is 80.

i.e., d = 80, Also a = 80

So, to find the interest at the end of 30 years, we shall find a_{30}

Now, $a_{30} = a + (30 - 1)d = 80 + 29 \times 80 =$ Rs.2400.

So, the interest at the end of 30 years will be Rs.2400.

3. The 17th term of an A.P. exceeds its 10th term by 7. Find the common difference. (P.S.)

A. In an A.P. 17th term $a_{17} = a + 16d$

10th term = $a_{10} = a + 9d$

Given $a_{17} = a_{10} + 7$

$a + 16d = a - 9d = 7$

$7d = 7$

$\therefore$ Common difference = d = 1.

4. How many terms of the A.P., 24, 21, 18,..... must be taken so that their sum is 78 ? (PS)

A. We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$6 = \frac{n}{2}[51 - 3n]$$

$$S_7 = \frac{7}{2}[2a + (7-1)(-20)] = 700$$

So, $78 = \frac{n}{2}[48 + (n-1)(-3)]$

(or) $3n^2 - 51n + 156 = 0$

$(n - 4)(n - 13) = 0$

n = 4 (or) 13.

Both values of n are admissible. So, the number of terms is either 4 or 13.

5. A sum of Rs.700 is to be used to given seven cash prizes to students of a school for their overall academic performance. If each prize is Rs.20 less than its preceeding prize, find the value of each of the prizes. (Connection)

A. Let the prizes be $a_1, a_2, a_3, a_4, a_5, a_6, a_7$.

Every prize money is Rs.20 less than its preceeding prize except the first one. So, $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are in A.P.

Common difference = $d = a_2 - a_1 = -20$

($\because$ a_1 is 20 more than a_2)

Given, the sum of all the prizes = Rs.700

$$2a + 6(-20) = 700 \times \frac{2}{7} = 200$$

$2a - 120 = 200$

$2a = 200 + 120 = 320$

$$\therefore \ a = \frac{320}{2} = 160$$

The value of the prizes will be as follows

$a = a_1 = 160$

$a_2 = 160 - 20 = 140$

$a_3 = 140 - 20 = 120$

$a_4 = 120 - 20 = 100$

$a_5 = 100 - 20 = 80$

$a_6 = 80 - 20 = 60$

$a_7 = 60 - 20 = 40$.

6. Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2. (PS) (AS_1).

A. In a G.P., 8th term $a_8 = ar^7 = 192$ .....(1)

substituting r = 2 value in (1), we get

$a(2)^7 = 192$

$a \times 128 = 192$

$$\therefore a = \frac{192}{128} = \frac{3}{2}$$

$$= \frac{3}{2} \times (2)^{11} = 3 \times 2^{10}$$

$\therefore$ 12th term $= a_{12} = ar^{11}$.

7. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, then what would be the bacteria at the end of second hour, at the end of the fourth hour ? Find out the bacteria at the end of nth hour ? (AS_4)

A. No. of bacteria originally = 30

From the data given,

No. of bacteria at the end of first hour = 30 2 = 60

No. of bacteria at the end of second hour = 60 2 = 120

No. of bacteria at the end of third hour = 120 2 = 240

No. of bacteria at the end of fourth hour = 240 2 = 480

Writing the above result in a form of progression we get,

30, 60, 120, 240, 480........

The above progression is clearly a G.P.

First term = $a_1 = 30$

common ration = r = 2

$\therefore$ No. of bacteria at the end of nth hour = $a_n = a.r^{n-1}$

$= (30)\ (2)^{n-1}$

8. Find the sum of the numbers which have '1' in their one's place between 50 and 350.

A. Numbers which have '1' in their ones place between 50 and 350 are

51, 61, 71, 81,....., 341.

They are in A.P.

First term = $a_1 = 51$

Common difference = d = 61 – 51 = 10

The last term of the above progression

$a_n = 341$

In an A.P.

$a_n = a + (n - 1)d$

341 = 51 + (n – 1) (10)

341 = 51 + 10n – 10

341 = 41 + 10n

341 – 41 = 10n

(or) 10n = 300

$\therefore n = \frac{300}{10} = 30$ $\left(\because S_n = \frac{n}{2}[a + a_n]\right)$ $\left(r = \frac{120}{60} = 2\right)$

The sum of the numbers as asked

$= \frac{30}{2}[51 + 341]$

= 15 [392]

= 5880.

ONE MARKS QUESTIONS

1. Is the series of numbers 0.2, 0.22, 0.222, 0.2222....... form an A.P. ? If so, what is the common difference ? (AS_4) *(Reasoning)*

A. Given, 0.2, 0.22, 0.222, 0.2222......

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

In all the above events, $a_{k+1} - a_k$ is not equal.

So the above given series doesn't represent A.P.

2. If x, x+2, x+6 are the three consecutive numbers in G.P., find the value of x ? (AS_4)

A. x, x + 2, x + 6 are the three consecutive numbers in G.P.

$$\therefore \frac{x+2}{x} = \frac{x+6}{x+2}$$

$(x + 2)^2 = x (x + 6)$

$x^2 + 4x + 4 = x^2 + 6x$

$x^2 + 4x - x^2 - 6x = -4$

$-2x = -4$

$2x = 4$

Verification : x, x + 2, x + 6

2, 4, 8 are in G.P.

3. Which term of 2, 8, 32........ G.P. becomes 512 ? (AS_1)

A. Here a = 2, , $a_n = 512$

In a G.P. $a_n = a.r^{n-1} = 512$

$2 \quad (4)^{n-1} = 512$

$2 \quad (2^2)^{n-1} = 2^9$

$$= \frac{2\sqrt{6}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{9}}$$

$2^{2n-1} = 2^9$

$2n - 1 = 9$

$2n = 9 + 1 = 10$

So, 5th term of the G.P. 2, 8, 32........ becomes 512.

4. Find out 8th term of the progression ? (AS_1)

A. Given

First term = (a_1) =

Common ration

$$= \frac{\sqrt{18}}{\sqrt{9}} = \sqrt{2}$$

8th term = $a_8 = a.r^{8-1} = (\sqrt{3}).(\sqrt{2})^7$.

5. An employee started his salary of Rs.3000. If his annual increment is Rs.150/-, then what is his salary in the 8th year ? (AS_4)

A. Starting salary = (a_1) = Rs.3000

Annual increminent = Rs.150

His salary will be as follows

3000, 3150, 3300..............

($\because a_n = a + (n-1)d$)

Here n = 8

Above series is in A.P.

Common difference = d = 150

His salary in 8th year = $a_8 = a + (8-1)d$

= a + 7d

= 3000 + 7(150)

= 3000 + 1050 = Rs.4050/-.

6. How many multiples of 6 lie between 1 and 40? Do they form an A.P.? If so, find the sum of them? (AS_4)

A. Multiples of 6 which lie between 1 and 40 are as follows

6, 12, 18, 24............. 40 terms

$a = 6,\ d = a_2 - a_1 = 12 - 6 = 6,\ n = 40$

$$\times$$
$$\therefore\ S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{40} = \frac{40}{2}[2 \times 6 + (40-1)(6)]$$

= 20 [12 + 39 × 6]

= 20 [12 + 234]

= 20   246

S_{40} = 4920

So the sum of the multiples that lie between 1 and 40 is 4920.

4 MARKS QUESTIONS

1. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increased uniformely by a fixed number every year, find (i) the production in the 1st year, (ii) the production in the 10th year (iii) the total production in first 7 years. (AS_1)

Sol. Since the production increases uniformely by a fixed number every year, the number of TV sets manufactured in 1st, 2nd, 3rd..... years will form an A.P.

Let us denote the number of TV sets manufactured in the nth year by a_n.

Then, $a_3 = 600$ and $a_7 = 700$

(or) a + 2d = 600

and a + 6d = 700

solving these equations, we get d = 25 and a = 550

Therefore, production of TV sets in the first year is 550.

ii) Now $a_{10} = a + 9d = 550 + 9 \quad 25 = 775$

So, production of TV sets in the 10th year is 775.

iii) Also

$$= \frac{7}{2}[1100 + 150] = 4375$$

Thus, the total production of TV sets in first 7 years is 4375.

2. In the geometric progressions 162, 54, 18.... and $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}$...... have their nth term equal. Find the value of n. (PS)

A. Given first G.P. : 162, 54, 18....

a = 162, [illegible]

nth term = $a_n = a.r^{n-1}$

Second G.P. : $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}, ...$

First term (a) ,

nth term = a_n

$$= \frac{2 \times 3^{n-1}}{81}$$

Given that the nth terms of two G.P.'s are equal.

$$\therefore \frac{162}{3^{n-1}} = \frac{2 \times 3^{n-1}}{81}$$

$2 \times 3^{n-1} \quad 3^{n-1} = 162 \quad 81$

$3^{2n-2} = 81 \times 81$

$3^{2n-2} = 3^4 \quad 3^4$

$3^{2n-2} = 3^8$

$2n - 2 = 8$

$2n = 8 + 2 = 10$

$\therefore$ n = 5.

3. If the sum of the first n terms of an A.P. is $2n + 3n^2$ then find the rth term. (AS_1).

A. The sum of first n terms in an A.P.

$S_n = 2n + 3n^2$

If n = 1, $S_1 = 2(1) + 3(1)^2 = 2 + 3 = 5$

$\therefore a_1 = 5$

If n = 2, $S_2 = 2(2) + 3(2)^2 = 4 + 12 = 16$

$a_2 = S_2 - S_1 = 16 - 5 = 11$

If n = 3, $S_3 = 2(3) + 3(3)^2 = 6 + 27 = 33$

$\therefore a_3 = 33 - 16 = 17$

$\therefore$ A.P. = 5, 11, 17,......

$a_1 = 5$, $d = a_3 - a_2 = 17 - 11 = 6$

In an A.P., $a_r = a + (r - 1)d$

$= 5 + (r - 1)(6)$

$= 5 + 6r - 6 = 6r - 1$.

4. The sum of the three terms in an A.P. is 15 and the sum of the squares of the first and last terms is 58. Find the numbers. (AS_1).

A. Let the three terms be a – d, a, a + d

The sum of three terms = 15

a – d + a + a + d = 15

3a = 15

So, sum of the squares of the first and last terms = 58

$(a - d)^2 + (a + d)^2 = 58$

$2(a^2 + d^2) = 58$

$(5)^2 + d^2 = 29$

$d^2 = 29 - 25 = 4$

Case - i) If d = +2 then these terms are 5–2, 5, 5+2

= 3, 5, 7

Case - ii) If d = –2 those terms are 5–(–2), 5, 5+(–2)

= 7, 5, 3.

OBJECTIVE TYPE QUESTIONS

1. In an A.P., first term is 100 and common difference is –2, then its 51 term. [ ]

   A) 2 B) 0 C) –2 D) –14

2. In an A.P., if mth term is n and nth term is m, then its first term is .... [ ]

   A) m + n –1 B) m – n + 1 C) m + n D) 0

3. Sum of the n terms in the progression 1, 3, 5.... is [ ]

   A) $n^2 + n$ B) $2n^2 + n$ C) $n^2 - n$ D) n^2

4. If x – y, ..., (x+y) are in A.P., then the term in the blank is ? [ ]

   A) x B) y C) x – 2y D) x + 2y

5. If the nth term of an A.P. is 2n + 1, then its common difference ? [ ]

   A) 1 B) 2 C) 3 D) n

6. If a, b, c are in A.P., 1d e 1 2 1 1 b = ,, ,....,.... bc 4 2ca c ba [ ]

   A) c – a B) $\frac{a+c}{2}$ C) D) a + c

7. If a, b, c are in G.P., then are in .... [ ]

   A) A.P. B) H.P. C) G.P. D) None of the above

8. The angles in a concave polygon are in A.P. The smallest angle is 100 and the biggest angle is 140 then the number of sides of that polygon will be [ ]

   A) 6 B) 8 C) 10 D) 11

9. Sides of a right angle triangle are natural numbers and are in A.P. Then the measure of one of its side is .... [ ]

   A) 22 B) 58 C) 81 D) 91

10. One of the 4^{th} Arithmetic means that lie between 3, 23 is .... [ ]

    A) 6 B) 8 C) 15 D) 21

Fill in the following blanks with suitable answers :

1. If the first term of an A.P. is –2, 10th term is 16, then 15th term ....
2. If a, b, c are in A.P., then b + c, c + a, a + b are in ....
3. If a, b, c are in A.P., then are in ....

4. Values of   are in .... progression.

5. If the sum of n terms in A.P. is   , then its 10th term ....

6. The sum of the first 100 natural numbers is ....

7. If $10^6 - 1$ is divided by 999, then the quotient will be ....

8. The first term of an A.P. is   , 5th term is   then the common difference ....

9. If a, b, c are in A.P. and G.P. then ....

10. In an A.P., if m times of mth term is equal to n times of nth term, then (m + n)th term ....

11. If n Arithmetic means are inserted between a, b then its common difference ....

12. Sum of n terms of 1, 3, 5, ...

13. In a G.P., every term has a .... with its preceeding term.

14. Arithmetic mean of a+2, a, a–2 is ....

15. 11th term of 1, 3, 5, 7, ... is ....

16. 4,8,16,32...... is an example of a .... progression. $4n^2 - 3n$ $\cot\frac{\pi}{4}$, $\cot\frac{\pi}{6}$

17. In a G.P., if nth term is $2(0.5)^{n-1}$ then its common ratio is ...., first term is ....

18. In a G.P., 6th term is 24 and 13th term is 3/16. Its 20th term is ....

19. If (x – 3b), (x + b), (x + 5b) are in A.P. then its common difference ....

20. Three terms are in G.P. Their product is 216 and sum is 21. Then the middle term is ....

MATCH THE FOLLOWING :

Group - A		Group - B
1. Common difference of A.P.	[ ]	A) n – 12
2. 13th term of an A.P. (n–1), (n+2), (n–3)....	[ ]	B) –29
3. If $t_n = (-1)^n.n^2$ then its 7th term	[ ]	C) 1/6
4. 10th term of progression 16, 11, 6,......	[ ]	D) 3
5. If k+2, 4k–6, 3k–2 are in A.P., the value of k is	[ ]	E) n – 13
		F) 49
		G) 2
		H) –49

Group - A		**Group - B**
1. If the first term of G.P. is 50, 4th term is 1350 then the common ratio	[ ]	I)
2. Common ratio of G.P. $\frac{x}{y}, \frac{1}{x}, \frac{y}{x^3}$ is	[ ]	J) 1/y
3. If a, b, c, d, e are in G.P. then ae =	[ ]	K) 1
4. If r < 1 then the sum of n terms in G.P.	[ ]	L) 3
5. If are in G.P. then x value	[ ]	M) bd
		N)
		O)
		P) 2C

$\frac{a(1-r^n)}{1-r}$ $\frac{a(r^n-1)}{r-1}$ $\frac{2}{7}, x, \frac{7}{2}$ ± 1 $\frac{1}{x^2}$ $\frac{-7}{2}$

CHAPTER - 7

CO-ORDINATE GEOMETRY

Weightage of Marks :

No. of 2 marks questions = 2×2 = 4

No. of 1 mark questions = 1   1 = 1

No. of 4 marks questions = 1   4 = 4

No. of bits = 8   1/2 mark = 4

Total = 13M

- The pioneer of co-ordinate geometry is a well known French Mathematician known as Rhene-Deskorde.
- Every student can follow / learn pin points which are mentioned at the end of the chapter : under the heading of 'what we have discussed'.

2 MARKS QUESTIONS AND ANSWERS

1. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle. (R.P.)

Sol. Let given points A = (5, –2), B = (6, 4) and C = (7, –2).

Let us apply the distance formula to find the distances AB, BC and AC such as

We have,

$$=\sqrt{37}$$

$$AB = BC = \sqrt{37}$$

$$\sqrt{36}$$

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=10$$

$$\overline{AB}=\sqrt{(6-5)^2+(4+2)}=\sqrt{1^2+6^2}$$

$$\overline{BC}=\sqrt{(7-6)^2+(-2-4)^2}$$

$$=\sqrt{1^2+(-6)^2}$$

$$=\sqrt{1+36}$$

$$\overline{AC}=\sqrt{(7-5)^2+(-2+2)^2}=\sqrt{2^2}=2$$

we observed that

∴ Given vertices being the Isosceles Triangle.

2. If the distance between two points P(2, –3) and Q (10, y) is 10 units, then find y-co-ordinate. (P.S.)

Sol. Given points P(2, –3), Q(10, y) and $\overline{PQ}=10$ (by problem)

Squaring on both sides

$64 + (y + 3)^2 = 10^2$

$(y + 3)^2 = 100 - 64$

$(y + 3)^2 = 36$

$y + 3 = \pm 6$

$y + 3 = 6$ (or) $y + 3 = -6$

$y = 6 - 3 \quad y = -6 - 3$

$y = 3 \qquad y = -9$

$\therefore$ y = –9 or 3.

3. Find the method of dividing the line segment joining A(–4, 0) and B (0, 6) into four equal parts.

Sol. Point 'P' divides AB in the ratio 1 : 3

Q divides AB in the ratio 1 : 1

R divides AB in the ratio 3 : 1

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

We apply section formula such as

4. Check whether the points (1, –1) (2, 3) and (2, 0) are collinear or not ? Verify ? (R.P.)

Sol. Given points (1, –1) (2, 3) (2, 0)

Area of Triangle

$$= \frac{1}{2}\left|1(3-0) + 2(0+1) + 2(-1-3)\right|$$

$\therefore$ Given vertices (points) are not lie on same line or not collinear points.

5. If the points (K, K), (2, 3) and (4, –1) are collinear, then find value of K ? (P.S.)

Sol. Given that the points A (K, K), B (2, 3) and C (4, –1) are collinear.

$\therefore \Delta ABC = 0$

$6k - 14 = 0$

$6k = 14$

6. Determine x so that 2 is the slope of the line through P(2, 5) and Q(x, 3). (P.S.)

Sol. Slope of the line passing through P (2, 5) and Q (x, 3) is 2.

Here $x_1 = 5$, $y_1 = 5$, $x_2 = x$, $y_2 = 3$

Slope of

(Given in the problem)

$$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = 0$$

$-2 = 2x - 4$

$\Rightarrow 2x = 2$

7. Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9). (Comm. & P.S.)

Sol. Let the required point on x-axis be P(x, 0) and given points A (2, –5) and B (–2, 9)

Distance between two points

$$= \sqrt{(2-x)^2 + (-5-0)^2}$$

$$= \sqrt{(2-x)^2 + 25}$$

$$PB = \sqrt{(-2-x)^2 + (9-0)^2}$$

$= \sqrt{x^2 + 4x + 4 + 81}$

But given that PA = PB

square on both sides

$x^2 - 4x + 29 = x^2 + 4x + 85$

$4x + 4x = 29 - 85$

$8x = -56$

∴ The co-ordinates of the required point are (–7, 0).

8. If the points (1, 2), (–1, b) and (–3, –4) are collinear, find the value of b ? (P.S. & Conne.)

Sol. Let the given points be A (1, 2), B (–1, b) and C (–3, –4)

we know that ΔABC = 0

(Given points are collinear)

[illegible]

$4b + 4 = 0$

$4b = -4$

1 MARK QUESTIONS AND SOLUTIONS

1. Find distance between (0, –3) and (0, –8) points ? and justify that the distance between two points on y-axis is . (R.P.)

Sol. The distance between two points (0, –3) and (0, –8)

Here $x_1 = 0$, $x_2 = 0$

$= \sqrt{0 + (-5)^2} = \sqrt{25} = 5$

The distance between the points $(0, y_1)$ and $(0, y_2)$ which are lie on y-axis

2. Are the points (3, 2) (–2, –3) and (2, 3) form a triangle ? (R.P.)

Sol. Let us apply the distance formula such as and let P (3, 2), Q (–2, –3), R (2, 3)

$$PQ = \sqrt{(-2-3)^2 + (-3-3)^2}$$

$$= \sqrt{(-5)^2 + (-5)^2}$$

$$= \sqrt{25+25} = \sqrt{50}$$

$$QR = \sqrt{(2+2)^2 + (3+3)^2} = \sqrt{4^2 + 6^2}$$

= 7 – 21 (approx.)

$$PR = \sqrt{(2-3)^2 + (3-2)^2}$$

$$= \sqrt{(-1)^2 + 1^2} = \sqrt{1+1} = \sqrt{2} = 1.41 \text{ units (approx.)}$$

[illegible]

Since the sum of any two of these distances is greater than the third distance, therefore, the points P, Q and R form a triangle and all the sides of triangle is unequal.

3. Find the centroid of the triangle whose vertices are (3, –5)m (–7, 4) and (10, –2) ? (P.S.)

Sol. The co-ordinates of the Centroid are $= \left(\dfrac{x_1 + x_2 + x_3}{3}, \ \dfrac{y_1 + y_2 + y_3}{3} \right)$

$\therefore$ The centroid is (2, –1).

4. Find the slope of the line segment joining the two points (0, 0) and ($\sqrt{3}$, 3) ? (P.S.)

Sol. Given points that (0, 0) and (, 3)

Slope of the line

$$= \frac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = \sqrt{3}$$

5. Define 'Points of Trisection' ?

Sol. The points which divide a line segment into three equal parts are said to be the Trisectional points of the line segment.

6. Prove that the points A(4, 2), B (7, 5) and C(9, 7) are collinear. (R & P)

Sol. Area of $\Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|4(5-7) + 7(7-2) + 9(2-5)|$

$= \frac{1}{2}|35 - 35|$

$= \frac{1}{2} \times 0$

= 0.

$\therefore$ All given points are collinear.

(The points lie on the same line are called collinear).

4 MARKS PROBLEMS & SOLUTIONS

1. Prove that (–4, –7), (–1, 2), (8, 5) and (5, –4) are vertices of a Rhombus. And find its area. (P.S.)

Sol. Let the given points are A(–4, –7), B(–1, 2), C(8, 5), D(5, –4)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \frac{1}{2}|d_1 d_2|$

Distance between two points

$AB = \sqrt{(-1+4)^2 + (2+7)^2}$

$= \sqrt{9+81} = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}$

$CD = \sqrt{(5-8)^2 + (-4-5)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$

$BC = \sqrt{(8+1)^2 + (5-2)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$

$AC = \sqrt{(8+4)^2 + (5+7)^2} = \sqrt{144+144} = \sqrt{288} = 12\sqrt{2}$

$AD = \sqrt{(5+4)^2 + (-4+7)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$

$BD = \sqrt{(5+1)^2 + (-4-2)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

Here the four sides of Quadrilateral ABCD are equal.

i.e., AB = BC = CD = DA and AC $\neq$ BD

$\therefore$ ABCD is a Rhombus

$\therefore$ Area of ABCD

$$= \frac{1}{2} \times 12\sqrt{2} \times 6\sqrt{2}$$

= 72 sq.units

2. Find in what ratio does the point (–1, 6) divides the line segment joining (–3, 10) and (6, –8).

Sol. Let the ratio that the point (–1, 6) divides the line segment is K : 1.

$$\therefore P(-1, 6) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$$

$$(-1, 6) = \left(\frac{K \times 6 + 1 \times (-3)}{K+1}, \frac{K(-8) + 1(10)}{K+1} \right)$$

$$-1 = \frac{6K-3}{K+1} \text{ And } 6 = \frac{-8K+10}{K+1}$$

$$D = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = (1, 0)$$

–K – 1 = 6K – 3   6K + 6 = –8K + 10

–K –6K = –3 + 1   6K + 8K = 10 – 6

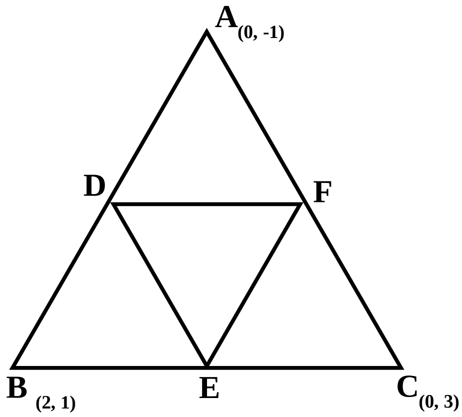
–7K = –2   14K = 4

$$K = \frac{-2}{-7} = \frac{2}{7} \qquad K = \frac{4}{14} = \frac{2}{7}$$

$\therefore$ Required ratio is 2 : 7.

3. Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle. (PS)

Sol.



Let D, E, F are the midpoints of the sides AB, BC and AC respectively.

Midpoint

$$E = \left(\frac{2+0}{2}, \ \frac{1+3}{2} \right) = (1, \ 2)$$

$$F = \left(\frac{0+0}{2}, \ \frac{-1+3}{2} \right) = (0, 1)$$

Area of a triangle $= \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_2) + x_3(y_2 - y_1) \right|$

Area of $\Delta ABC = \frac{1}{2}\left| 0(1-3) + 2(3+1) + 0(-1-1) \right|$

= 4 sq.units

Area of $\Delta DEF = \frac{1}{2}\left| 1(2-1) + 1(1-0) + 0(0-2) \right|$

= 1 sq.unit

Ratio of the areas = $\Delta ABC : \Delta DEF$

= 4 : 1.

4. Find the area of a triangle formed by (8, –5), (–2, 7) and (5, 1) by Heron's Formula. (PS)

$= \frac{1}{2}|1+1| = \frac{1}{2}|2|$

Sol. The given points are A (8, –5), B (–2, 7), C(5, 1)

Length of AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-3)^2 + (7+5)^2}$

$= \sqrt{100+144} = \sqrt{244} = 15.62$

$BC = \sqrt{(5+2)^2 + (1-7)^2}$

$= \sqrt{49+36} = \sqrt{85} = 10.63$

$AC = \sqrt{(5-8)^2 + (1+5)^2}$

$= \sqrt{9+36} = \sqrt{45} = 6.7$

$S = \frac{AB + BC + AC}{2}$

$= \frac{15.62 + 10.63 + 6.7}{2}$

$= \frac{32.95}{2} = 16.475$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16.475 \times 0.855 \times 5.845 \times 9.775}$

$= \sqrt{804.809}$

= 28.37 sq.units.

5. Find the area of the quadrilateral formed with the points (3, –5), (5, –1), (2, 1) and (–3, –2).

Sol. Area of a triangle,

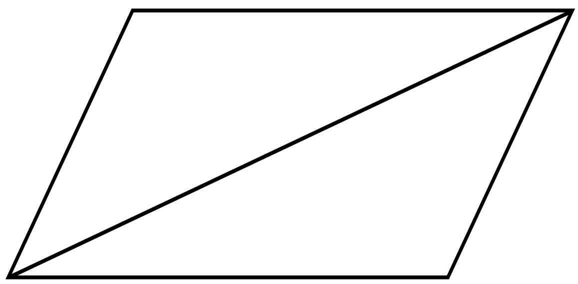
$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_2) + x_3(y_2 - y_1)\right|$$

Area of $\Delta ABC = \frac{1}{2}\left|3(-1-1) + 5(1+5) + 2(-5+1)\right|$

= 8 sq.units

Area of $\Delta ACD = \frac{1}{2}\left|3(1+2) + 2(-2+5) - 3(-5-1)\right|$

$= \sqrt{16.475(16.475-15.62)(16.475-10.63)(15.475-6.7)}$



$= \frac{1}{2} \times 33$ = 16.5 sq.units

Area of the quadrilateral

ABCD = ΔABC + ΔACD

= 8 + 16.5

= 24.5 sq.units.

Multiple Choice Questions :

1. One end of diameter is (–3, 4) and centre of the circle is (0, 0) the other end co-ordinates is .....

A) (4, –3) B) (3, 4) C) (3, –4) D) (–4, –3) ( )

2. Slope of the line 3x – 4y + 12 = 0 is .... ( )

A) B) C) 4 D)

3. Area of the triangle with the vertices A(0, 0), B(0, 3) and C(4, 0) is .... ( )

   A) 12 B) 5 C) 6 D) 7

4. The ratio that y-axis divides the joining the points (5, 7) and (–1, 3) is......... ( )

   A) 5 : 1 B) 3 : 1 C) 2 : 1 D) 4 : 3

5. Intersection point of the diagonal of parallelogram with vertices (2, 3) (3, 4) (6, 9) and (5, 8) is ....

   A) B) (4, 6) C) D) ( )

6. (2, –1) is the mid-point of a segment, if one end point of the segment is (5, 3) then the other end point is .... ( )

   A) (7, 2) B) (3, 4) C) (–1, –5) D) (3, 2)

7. The midpoint of the joining of (–4, a) and (2, 8) is (–1, 5) then the value of a is ...... ( )

   A) 4 B) 3 C) 2 D) 1

8. The point on y-axis that is equidistant from (2, 1) and (4, 5) is .... ( )

   A) (0, 9) B) (0, 2) C) (0, 9/2) D) (0, 1)

9. Two vertices of a triangle are (–4, 6), (2, –2) and its centroid is (0, 3) then third vertex is ....

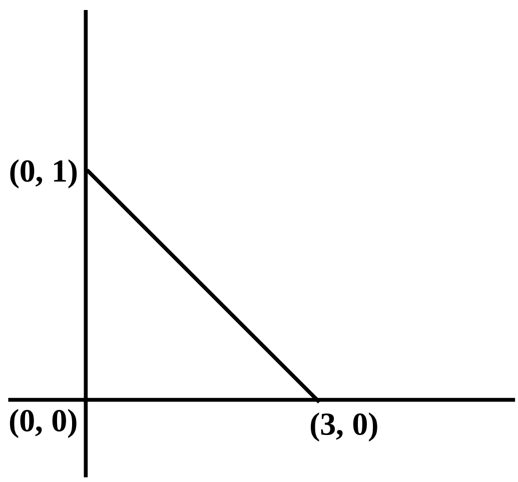
   A) (4, –6) B) (–2, 2) C) (–2, 5) D) (2, 5) ( )

10. One of the trisection point of joining the points (2, 3) and (6, 5) is .... ( )

   A) B) C) $\left(\frac{10}{3}, \frac{11}{3}\right)$ D) (10, 11)

Bits :

11. If AB + BC = AC then the points A, B, C are ....
12. If (1, 2), (–3, 4), (7, –K) are collinear then K = ...
13. Slope of the line ax + by + c = 0 is ....
14. The distance between (2, K) and (4, 3) is 8 then the value of K is ....
15. One end of diameter of a circle with the centre (0, 0) is (4, 5) ....
16. The ratio that the point (4, 5) divides the joining of (2, 3) and (7, 8) is ....
17. Slope of the line in the adjacent figure is ....



18. Two vertices and centroid of a triangle are (6,4) (3,2) and (5,0) respectively. Then third vertex is ....

19. A(p, 2), B(–3, 4), C(7, –1) lie on the same line then the value of p is..........

20. Father of co-ordinate geometry is ....

21. The line equation that bisects the 1st quadrant in the rectangular system is ....

22. The line y = x passes through ....

23. The point that the line 2x + 3y = 9 cuts the y-axis is ....

24. The intersection point of x = 3 and y = –2 is ....

25. The ratio that the x-axis divides the joining of (3, 6) and (12, –3) is ....

26. Centroid of the triangle with vertices (–4, 4), (–2, 2) and (6, 12) is ....

27. If a is negative integer then (a, –a) lies in .... quadrant.

28. Distance of the point (–4, 3) from x - axis is ....

29. The distance of a point (2, 3) on the circle of centre (0, 0) is ....

30. Angle between x and y axis is ....

31. Slope of x - axis is ....

32. Slope of y - axis is ....

33. Co-ordinates of mid points joining of (x_1, y_1) and (x_2, y_2) is ....

34. Slope of the line joining the points (5, –1) and $\sqrt{8}$ (0, 8) is ....

Match the following :

1.1.	Slope of line passing through (0, 2) and (4, 0)	[ ]	A) 4
2.	Area of triangle formed by (0, 0) (3, 0) (0, 3)	[ ]	B) (0, –c/b)
3.	Distance of (5, 2) (3, k) is , value of K	[ ]	C) –1/2
4.	Point where the line ax+by+c = 0 cuts y-axis is	[ ]	D) (–c/m, 0)
5.	y = mx + c cuts the x-axis at	[ ]	E) 1/2
			F) 0 or 4
			G) 9/2
			H) (–c/b, 0)
2.1.	Radius of a circle with centre (0, 0) is 3 units then point (2, 3) is at	[ ]	L) (0, 2)
2.	Mid point of the joining of (–1, 4) and (2, –2) is	[ ]	M) outside of circle
3.	Point of intersection of x – 2y = 4 and x + y = –2 is	[ ]	N) square
4.	Quadrilateral formed with (0, 0) (1, 0) (1, 1)	[ ]	O) 45^0

and (0, 1) is a ....

5. Angle made by x + y = 0 with x-axis is .... [ ] P) inside of circle

Q)

R) 135^0

S) (0, –2)

BITS - ANSWERS

11. collinear	12. 1	13. –a/b	14.
15. (–4, –5)	16. 2 : 3	17. –1/3	18. (6, –6)
19. 1	20. Renedecarte	21. y = x	22. origin
23. (0, 3)	24. (3, –2)	25. 2 : 1	26. (0, 6)
27. Q_2	28. 3	29. $\sqrt{13}$	30. 90^0
31. 0	32. undefined	33.	34. .

....

$$\left(\frac{x_1 + x_2}{2}, \frac{x_2 + y_2}{2}\right) \quad \frac{3 \pm \sqrt{15}}{5} \quad \left(\frac{-1}{2}, \frac{1}{2}\right) \quad 2\sqrt{2}$$

S.S.C. X CLASS

MODEL PAPER - I

MATHEMATICS (E.M.), PAPER - I

Time : $2^1/_2$ Hrs.] **PARTS - A & B** **Max. Marks : 50**

Instructions :

1. **Answer the questions under Part - A on a separate answer book.**
2. **Write the answers to the questions under Part - B on the Questions Paper itself and attach it to the answer book of Part - A.**

Time : 2 Hrs.] **PARTS - A** **Marks : 35**

SECTION - I (Marks : 5 x 2 = 10)

Note : 1) Answer any FIVE questions, choosing atleast Two from each of the following two groups i.e. A and B.

2) Each questions carries 2 Marks.

Group - A

1. Express the number 3825 as a product of its prime factors.
2. If A = {x : x is a multiple of 10}, B = {10, 15, 20, 25, 30.....} then state whether A = B or not.
3. If sum of the zeroes of the polynomial $Kx^2 - 3x + 1$ is '1', Find the value of 'K' ?
4. Find the roots of $2x^2 + x - 6 = 0$.

Group - B

5. 10 students of class - X took part in a mathematics quiz. If the number of girls is 4 more than number of boys then find the number of boys and the number of girls who took part in the quiz.
6. A sum of Rs.700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs.20 less than its preceeding prize, find the value of each of the prizes.
7. Find the values of y for which the distance between the points P (2, –3) and Q (10, y) is 10 units.
8. Verify whether the folowing points are collinear. (1, –1), (2, 3), (2, 0).

SECTION - II

Note : 1) Answer any FOUR of the folling six questions.

2) Each question carries 1 Mark.

9. Write the following in logarithmic form $3^5 = 243$.
10. If A = {1, 2, 3, 4}, B = {2, 4, 6, 8} then, Find A – B ?
11. If A = {1, 4, 9, 16, 25}, Write set-builder form ?
12. If $P(x) = 2x^3 + x^2 - 5x + 2$, then find P(O) ?
13. 0.2, 0.22, 0.222, 0.2222........... is it in A.P.? If it is an A.P., then find the common difference 'd' ?
14. If A (2, 1), B (2, 6). Justify that the line segment formed by given points is parallel to y-axis ? What can you say about their slope ?

SECTION - III (Marks : 4 x 4 = 16)

Note : 1) Answer any FOUR questions, choosing TWO from each of the following groups. i.e., A and B.

2) Each question carries 4 marks.

Group - A

15. Prove that is irritational.

16. If A = {4, 5, 6}, B = {7, 8} then show that (i) $A \cup B = B \cup A$, (ii) .

17. Verify that 3, –1, –1/3 are the zeroes of the cubic polynomial $P(x) = 3x^3 - 5x^2 - 11x - 3$, and then verify the relationship between the zeroes and the co-efficients.

18. Find the roots of the equation .

Group - B

19. Solve the equations

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4} \text{ and } \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}.$$

20. Compare the following pair of linear equations and fill up the blanks.

Pair of Lines	$\frac{a_1}{a_2}$			Comparison of ratios	Graphical representation	Algebraic interpretation
1. 5x – 2y + 4 = 0 10x – 4y + 8 = 0	......				.....	infinite no. of solutions
2. x + 3y – 5 = 0 5x – 2y – 6 = 0		....			Intersecting lines	....
3. 6x – 7y + 3 = 0 6x – 7y + 5 = 0	....	$\frac{7}{7}$	......	....	......	No solution

$A \cap B = B \cap A$ $\quad \frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, \ (x \neq -4, 7)$

21. Appa Rao started work in 1995 at an annual salary of Rs.5000 and received an increment of Rs.200 each year. In which year did his income reach Rs.7000 ?

22. Find the area of the quadrilateral whose vertices, taken inorder, are (–4, –2), (–3, –5), (3, –5) and (2, 3).

SECTION - IV

23. Draw the graph of the polynomial $P(x) = x^2 - 3x - 4$ and find the zeroes. Justify the answer.

24. Solve the given pair of equations graphically 5x + 7y = 50 and 7x + 5y = 46.

PART - B

Time : 30 min **Marks : 15**

Note : 1. Each question carries 1/2 mark.

2. Answers are to be written in the question paper only.

3. All questions are to be answered.

4. Marks willnot be given for over written, re-written or erased answers.

I. Write the Capital Letter of the correct answer in the brackets provided against each question.

1. If the H.C.F. of the two numbers 26, 169 is 13 then, the L.C.M. of the two numbers is ......
   A) 26 B) 52 C) 338 D) 368 ( )
2. If $P(x) = 7x^2 - 3x^2 + 1$, then coefficient of x^0 .... ( )
   A) 0 B) 1 C) –3 D) 7
3. If $L_1 = 2x + 2y - 8 = 0$ and $L_2 = x + y - 4 = 0$ are coincident lines and $L_1 = KL_2$, then find the value of K = ... ( )
   A) B) 2 C) 1 D) 1/2
4. Which true for Arithmetic Progression ? ( )
   A) $a_n = S_n + S_{n-1}$ B) $a_n = a + (n-1)d$ C) $S_n = n[2a+(n-1)d]$ D) All the above
5. From adjacent figure, x = ... ( )
   A) 5 B) 7 C) 12 D) 25
6. The pair of inconsistent equations are .... $\frac{1}{8}$ ( )
   A) intersecting B) parallel C) coincident D) none
7. The slope of x-axis .... ( )
   A) 0 B) –1 C) +1 D) not defined
8. If (0, 0), (a, 0), (o, b) are collinear, then ( )
   A) ab = 0 B) a = b C) a = –b D) None
9. If the end points of a diameter are (–2, 8) and (6, –4) then the centre of the circle is .... ( )
   A) (3, 6) B) (4, 2) C) (2, 2) D) (–3, 2)
10. The distance between the y-axis and the point (–8, –7) is .... ( )
    A) 8 B) –7 C) –8 D) 7

Fill up the blanks :

11. No. of zeroes in the number $n = 2^3 \times 3^4 \times 5^4 \times 7$ ....
12. If $b^2 - 4ac < 0$, then the roots of $ax^2 + bx + c$ are ....
13. The degree of the linear polynomial is ....
14. The solution of the pair of equations x + y = 14, x – y = 4 is ....
15. If (2x + 3) (x – 1) = 0 then x = ... or ....
16. x, (x+2), (x+6) are three consecutive numbers of a GP then x = ...
17. If the distance between two points (2, 8) and (2, K) is 3, then K = ...
18. The centroid of a triangle divides each median in the ratio of ....

19. The distance between the two points (a cosθ, 0), (0, a sinq) is ....

20. (–2, 8) belongs to .... Quadrant.

III. Match the following. **10 x $^1/_2$ = 5**

i. Group - A **Group - B**

21. If $\log_{10} 0.0001 = x$ then x = ... [ ] A) $\log_a xy$

22. [ ] B) 10

23. $\log_a x + \log_a y$ = ... [ ] C) 1

24. $\log_{2015} 2015$ [ ] D) Recurring decimal

25. is a .... [ ] E) –4

F) 2015

G) $\log_a (x + y)$

H) non-recurring decimal

ii. Group - A **Group - B**

26. Cubic Polynomial [ ] A) $a^2 - 4bc$

27. Speed = ... [ ] B)

28. Discriminent of $bx^2 + ax + c = 0$ is .... [ ] C)

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n$ $\log_{\sqrt{2}} 32 = \dots$ $\frac{}{2\ 2}$

29. is .... [ ] D) $ax^3 + bx^2 + cx + d$

30. If $P(x) = 2^{-x}$, then P(1) = ... [ ] E) $b^2 - 4ac$

F) n^{th} degree polynomial

G) distance / time

H) distance x time

S.S.C. X CLASS

MODEL PAPER - II

MATHEMATICS (E.M.), PAPER - I

Time : $2^1/_2$ Hrs.] **PARTS - A & B** **Max. Marks : 50**

Instructions :

1. **Answer the questions under Part - A on a separate answer book.**
2. **Write the answers to the questions under Part - B on the Questions Paper itself and attach it to the answer book of Part - A.**

Time : 2 Hrs.] **PARTS - A** **Marks : 35**

SECTION - I (Marks : 5 x 2 = 10)

Note : 1) Answer any FIVE questions, choosing atleast Two from each of the following two groups i.e. A and B.

2) Each questions carries 2 Marks.

Group - A

(Real Numbers, Sets, Polynomials, Quadratic Equations)

1. Without actually performing division, state whether $\frac{16}{25}$ will have aterminating decimal form or a non-terminating, repeating decimal form.
2. If A = {2, 3, 4, 5} ; whether , A are equal sets ? Justify your answer.
3. If 1/2 is a zero of the polynomial $2x^2 + 3x + \lambda$, then find values of λ and another zero of the polynomial ?
4. Find two numbers whose sum is 27 and product is 182.

Group - B

(Pair of Linear Equations in two variables, Progressions, Co-ordinate Geometry)

5. If the larger of two supplementary angles exceeds the smaller by 25^0, to find out the angles, form a pair of linear equations.
6. Which terms of the A.P. 21, 18, 15,..... is '–81' ? Is there any term '0' ?
7. Find the co-ordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).
8. Find the area of a triangle whose vertices are A(5, 2) B (4, 7) and C (7, –4).

SECTION - II

Note : 1) Answer any FOUR of the folling six questions.

2) Each question carries 1 Mark.

9. Expand log 100.
10. If A = {1, 3, 5, 7}, B = {1, 2, 3, 4} then find ?

11. Give one example for Empty Set ?

12. Find a quadratic polynomial if its zeroes are –2 and +3 ?

13. How many three digit numbers are divisible by 7 ?

14. Where do these points lie in a co-ordinate plane : (–4, 0), (2, 0), (6, 0), (–8, 0).

SECTION - III (Marks : 4 x 4 = 16)

Note : 1) Answer any FOUR questions, choosing TWO from each of the following groups. i.e., A and B.

2) Each question carries 4 marks.

Group - A

(Real Numbers, Sets, Polynomials, Quadratic Equations)

15. Prove that .

16. If A = {2, 4, 6, 8, 10} and B = {3,6, 9, 12, 15} then find $A \cup B$, $A \cap B$, $A - B$ and $B - A$?

17. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are ?

18. Find the roots of the equation $\frac{1}{x} - \frac{1}{x-2} = 3 \quad (x \neq 0, Z)$.

Group - B

(Pair of Linear Equations in two variables, Progressions, Co-ordinate Geometry)

19. Solve the following linear equations

$\log_a xy = \log_a x + \log_a y$

$\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\frac{2}{x} + \frac{3}{y} = 13$ and $\frac{5}{x} - \frac{4}{y} = 2$.

20. Check whether the following equations are consistent or inconsistent.

i) x + 5y – 4 = 0,   2x + 10y – 8 = 0

ii) 4x – y + 5 = 0,   12x – 3y – 7 = 0

21. A sum of Rs.1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an A.P. ? If so, find the interest at the end of 30 years.

22. If (1, 2) (4, y) (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

SECTION - IV (1 x 5 = 5)

(Polynomials, Pair of Linear Equations in two variables)

23. Draw a graph of y = x2 – x – 6 and find its zeroes. Justify your answer.

24. Solve the following linear equations through the graph x + 2y = –1 and 2x – 3y = 12.

PART - B

Time : 30 min **Marks : 15**

Note : 1. Each question carries 1/2 mark.

2. Answers are to be written in the question paper only.

3. All questions are to be answered.

4. Marks willnot be given for over written, re-written or erased answers.

I. Write the Capital Letter of the correct answer in the brackets provided against each question.

1. is a .... ( )

   A) Intezer Number B) Rational Number C) Irrational D) None of these

2. If , then quadratic polynomial is .... ( )

   A) $4x^2 + 3x - 1$ B) $x^2 - 3x - 1$ C) $4x^2 - 3x + 1$ D) $x^2 + 2x + 1$

3. One of following statement is false which is related the pair of equations and . ( )

   A) Consistent Equations B) x = 0, y = 0

   C) parallel lines D) unique solution

4. If sum of 'n' terms in a A.P. is , d is denoted for .... ( )

   A) first term B) common difference [illegible] C) common ratio D) Diameter

5. If $\frac{x}{a-b} = \frac{a}{x-b}$, then x = ... ( )

   A) a – b or a^2 B) b – a or a C) a – b or a/3 D) b + a or a/2

6. The equation of a line which intersect at (0, –4) of y-axis is .... ( )

   A) x – 4 = 0 B) x + 4 - 0 C) y + 4 = 0 D) y – 4 = 0

7. The slope of a line 3x – 4y + 12 = 0 is ( )

   A) B) C) 4 D)

8. The point of y-intercept by a equation of line ax – by – c = 0 is .... ( )

   A) B) C) D) (0, –C)

9. Pair of lines : y = 2x – 3 ; y = 2x – 4 are .... ( )

   A) perpendicular B) intersected C) parallel D) coincide lines

10. The angle of equation of line : with x-axis in the positive direction is .... ( )

    A) 45^0 B) 60^0 C) 90^0 D) 30^0

Fill up the blanks :

11. Decimal form of is ....

12. If $A \cap B = \phi,\ n(A \cup B) = 7,\ n(A) = 7$ then n(B) = ...

13. If p(x) = $x^2 - x - 2$ is a polynomial, then p(1) + p(0) = ...

14. A pair of linear equations is depedent and have .... solutions.

15. The quadratic equation involved in the (2x – 1) (x – 3) = (x + 5) (x – 1) is ....

16. The list of numbers 4, 8, 16, 32 are in .... progressions.

17. The distance between origin and (0, 10) points is ....

18. Ratio of the line joined by the points (8, 6) and (0, 10) is divided by the another point (4, 8) is ....

19. The length of diagonal of Rectangle AOBC in which vertices having A(4, 0), B(4, 3), C(0, 3), O(0, 0) is .... units.

20. The point (a, –a) lie in .... quadrant if a < 0.

III. Match the following. **10 x $^1/_2$ = 5**

i. Group - A **Group - B**

21. $\log_7 1$ = ... [ ] A) 1
22. logarithmic form of $10^{-3} = 0.001$ ..... [ ] B) 3 log 2
23. log 16 – log 2 = ... [ ] C) –3
24. If $\log_3 \frac{1}{27} = y$ then y = ... [ ] D) 0
25. The number is .... [ ] E) recurring decimal

F) $\log_{10} 0.001 = -3$
G) $\log_{10} 0.01 = -3$
H) terminating decimal

ii. Group - A **Group - B**

26. Degree of p(x) [ ] A) It is quadratic polynomial
27. Sum of the coefficients are [ ] B) 2
28. Sum of the zeroes [ ] C) It is linear polynomial
29. Product of the zeroes [ ] D) 3
30. If coefficient of x^3 is zero, then [ ] E) –4

F) –2
G) 4

S.S.C. X CLASS

MODEL PAPER - III

MATHEMATICS (E.M.), PAPER - I

Time : $2^1/_2$ Hrs.] **PARTS - A & B** **Max. Marks : 50**

Instructions :

1. **Answer the questions under Part - A on a separate answer book.**
2. **Write the answers to the questions under Part - B on the Questions Paper itself and attach it to the answer book of Part - A.**

Time : 2 Hrs.] **PARTS - A** **Marks : 35**

SECTION - I (Marks : 5 x 2 = 10)

Note : 1) Answer any FIVE questions, choosing atleast Two from each of the following two groups i.e. A and B.

2) Each questions carries 2 Marks.

Group - A

(Real Numbers, Sets, Polynomials, Quadratic Equations)

1. Find LCM and GCD of 76 and 108 by prime factorization method.
2. If A = {Quadrilaterals}, B = {Squares, Rectangles, Trapezium, Rhombus} then verify is $A \subset B$ or $B \subset A$? Justify your answer.
3. If $p(x) = x^3 - 1$ then find the value of p(1), p(–1), p(0) and p(2).
4. Find the value of K for quadratic equation $2x^2 + kx + 3 = 0$ so that it has two equal roots.

Group - B

(Pair of Linear Equations in two variables, Progressions, Co-ordinate Geometry)

5. One of the complementary angles is 20^0 more than the other. Write the equatins to find the angles.
6. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally. Then what would be the number of bacteria in second hour, fourth hour and nth hour ?
7. Find the coordinates of the point that divides the joining of (–1, 7) and (4, –3) in the ratio of 2 : 3.
8. Can we construct a triangle with the points (1, 5), (5, 8) and (13, 14) ? Write the reason.

SECTION - II

Note : 1) Answer any FOUR of the folling six questions.

2) Each question carries 1 Mark.

9. 2 log 3 – 3 log 2, write this as single log.
10. Find the cardinal number of the set A = {x, y, z, p, q}.
11. Write two examples for disjoint sets.

12. If α, β, γ are the zeroes of $p(x) = 4x^3 + 8x^2 - 6x - 2$ then find the value of $\alpha\beta + \beta\gamma + \alpha\gamma$.

13. Is 550, 605, 665.5,......... form G.P. ? If so find its common ratio.

14. If A (3, 2), B (–8, 2) are the points on a line $\overrightarrow{AB}$ then find slope of the line. When the line parallel to x-axis and why ?

SECTION - III (Marks : 4 x 4 = 16)

Note : 1) Answer any FOUR questions, choosing TWO from each of the following groups. i.e., A and B.

2) Each question carries 4 marks.

Group - A

(Real Numbers, Sets, Polynomials, Quadratic Equations)

15. Prove that $\sqrt{2}$ is an irrational number.

16. If A = {3, 6, 9, 12, 15, 18, 21} and
B = {4, 8, 12, 16, 20} then
find A – B, B – A, B – D, C – A ?

17. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were (x – 2) and –2x + 4 respectively. Find g(x).

18. Find the roots of the equation $\frac{x-1}{x+2} + \frac{x-3}{x-4} = \frac{10}{3} \; (x \neq -2, 4)$.

Group - B

(Pair of Linear Equations in two variables, Progressions, Co-ordinate Geometry)

19. Solve the following equations by dimination method,

$$x + \frac{6}{y} = 6 \quad \text{and} \quad 3x - \frac{8}{y} = 5.$$

20. Verify the following pair of equations are consistent or inconsistent and solve them
2x – 5y + 6 = 0 and 4x + 2y – 12 = 0.

21. A lader consists 25 steps. The length of steps are uniformly decreasing from bottom to top. If the length of bottom step is 45cm and that of the top is 25cm. And the distance between these two steps is $2^1/_2$cm. And the distance between these two steps is $2^1/_2$m. The what is the length of the wood to prepare all the steps.

22. Find the raio of areas of a triangle with the vertices (0, –1), (2, 1) and (0, 3). And the triangle formed with the midpoints of the sides of the triangle.

SECTION - IV (1 x 5 = 5)

(Polynomials, Pair of Linear Equations in two variables)

23. Draw the curve $y = 6 - x - x^2$ and write its zeroes. What did you notice.

24. Solve the following equation by graphical method. x + 3y = –4 and 2x – y = 6.

PART - B

Time : 30 min **Marks : 15**

Note : 1. Each question carries 1/2 mark.

2. Answers are to be written in the question paper only.

3. All questions are to be answered.

4. Marks willnot be given for over written, re-written or erased answers.

I. Write the Capital Letter of the correct answer in the brackets provided against each question.

1. Index of z when 144 is expressed as product of primes. ( )

A) 4 B) 5 C) 6 D) 3

2. In the given below which graph shows different solutions of a quadratic equation ( )

A) B) C) D)

3. If kx + 2y – 5 = 0 and 6x + 4y + 6 = 0 coincide each other then value of K is .... ( )

A) 12 B) 6 C) 5 D) 3

4. Which term of general G.P. is $a.r^n$. ( )

A) $(n+2)^{th}$ B) $(n-1)^{th}$ C) $(n+1)^{th}$ D) n^{th}

5. In a cubic polynomial if there is no x term then .... ( )

A) $\alpha + \beta + \gamma = 0$ B) $\alpha\beta + \beta\gamma + \alpha\gamma = 0$ C) $\alpha\beta\gamma = 0$ D) None

6. In the adjacent diagram the intersection point of the lines is ..... ( )

A) (–2, 0) B) (2, 0) C) (0, –2) D) (1, 2)

7. The slope of y - axis is .... ( )

A) 1 B) –1 C) 0 D) undefined

8. Two vertices of a triangle are (3, 5), (–4, –5) and its centroid is (4, 3) then the third vertex is ....

A) (13, 9) B) (–9, –13) C) (9, 13) D) (13, –9)

9. The angle between x and y axis is .... ( )

A) 0^0 B) 180^0 C) 360^0 D) 90^0

10. (–3, 0), (0, 5), (3, 0) are the vertices of .... triangle. ( )

A) Scalene B) Isosceles C) Equilateral D) Right

Fill up the blanks :

11. p/q form of 0.4 is ....

12. Number of elements in an empty set is ....

13. The conditions that the roots of $ax^2 + bx + c =$ ar complex numbers is ....

14. If a, x + b, y + c = 0 and $a_2x + b_2y + c_2 = 0$ have only one solution then relation between coefficients is ....

15. A real number 'K' is a .... of p(x) ; if p(k) = 0.

16. The sum of first 10 natural numbers is ....

17. If the distance between (3, k) and (4, 1) is then K = ...

18. If A, B, C are collinear the area of ΔABC is ....

19. The co-ordinates of mid-point of P(x,y) and $Q(x_2,y_2)$ is ....

20. The slope of a line joining (5, –1) and (0, 8) is ....

III. Match the following. **10 x $^1/_2$ = 5**

i.

Group - A		Group - B
21. value of 0.01	[ ]	A) log 30 – log 2
22. logarithmic form of $x^0 = 1$	[ ]	B) 6
23. log 3 + log 5 = ...	[ ]	C) Rational Number
24. If then x = ...	[ ]	D) –2
25. is a	[ ]	E) $\log_1 x = 0$
		F) –3
		G) $\log_x 1 = 0$
		H) Irrational Number

ii.

Group - A		Group - B
26. $\alpha^2 + \beta^2$	[ ]	A) 2
27. The degree of	[ ]	B) 0
28. Sum of the zeroes of $P(x) = y^3 - 1$	[ ]	C) $ax^2 + bx + c = 0$
29. General form of quadratic equation is	[ ]	D) $(\alpha + \beta)^2 - 2\alpha\beta$
30. If $\alpha + \beta = -1$, $\alpha\beta = 2$, the quadratic equation is	[ ]	E) ax + b
		F) $x^2 + x + 2$
		G) $(\alpha - \beta)^2 + \alpha\beta$

$\log 729 = x$ $\sqrt{10}$ $x^7 + 7x - 1$

S.S.C. X CLASS

MODEL PAPER - IV

MATHEMATICS (E.M.), PAPER - I

Time : $2^1/_2$ Hrs.] **PARTS - A & B** **Max. Marks : 50**

Instructions :

1. **Answer the questions under Part - A on a separate answer book.**
2. **Write the answers to the questions under Part - B on the Questions Paper itself and attach it to the answer book of Part - A.**

Time : 2 Hrs.] **PARTS - A** **Marks : 35**

SECTION - I (Marks : 5 x 2 = 10)

Note : 1) Answer any FIVE questions, choosing atleast Two from each of the following two groups i.e. A and B.

2) Each questions carries 2 Marks.

Group - A

(Real Numbers, Sets, Polynomials, Quadratic Equations)

1. write in decimal form.
2. If A = {1, 2, 3, 4, 5}, B = {2, 4, 5} then verify A – B, B – A are disjoint sets or not.
   A23 B $2^3 5^2$
3. Find the area of rectangle whose length and breadth are the roots of the quadratic equation $x^2 - 6x + 8 = 0$.
4. Find the discriminant of the quadratic equation $6x^2 - 2x + 5 = 0$ and hence find the nature of roots.

Group - B

(Pair of Linear Equations in two variables, Progressions, Co-ordinate Geometry)

5. Formulate a pair of equations to solve "5 books and 8 pens together costs 2.115. Also cost of 6 books and 5 pens together costs 2.115".
6. In a nursery 1st row contains 17 rose plants, 2nd row contains 14 plants, 3rd row contains 11 plants. And in the last row there are 2 plants. How many rows are there in the nursery.
7. Centroid of a triangle with vertices (2, 3), (x, y), (3, –2) is the origins. Then find (x, y).
8. Can you draw a triangle with (3, 2), (–2, –3) and (2, 3). Justify your answer.

SECTION - II

Note : 1) Answer any FOUR of the folling six questions.

2) Each question carries 1 Mark.

9. Find the value of $\log_3 81$.
10. A = {2, 4, 6, 8}, B = {2, 4, 8, 16} then find .
11. Give an example for 'infinite set'.

12. Find the remainder when $x^4 - 3x^3 - 5x^2 - 6x + 7$ is divided by $x - 1$.

13. Cost of digging a well for first metre is Rs.150 and for rest Rs.50 per meter. The cost of digging for 1st meter, 2nd meter, 3rd meter,........... form an A.P? or not and why ?

14. End points of a segment are (2, 3) and (4, 5). Find the slope of segment.

SECTION - III (Marks : 4 x 4 = 16)

Note : 1) Answer any FOUR questions, choosing TWO from each of the following groups. i.e., A and B.

2) Each question carries 4 marks.

Group - A

(Real Numbers, Sets, Polynomials, Quadratic Equations)

15. Prove that   is an irrational number.

16. If A = {x : x is a natural number}, B = {x : x is even natural number}, C ={x : x is odd natural number}, D = {x : x is a prime number} then Find $A \cap B,\ B \cap C,\ B \cap D,\ C \cap D$.

17. Does $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$?And verify it.

18. Solve the quadratic equation $x^2 + 7x - 6 = 0$ by completing the square.

Group - B

(Pair of Linear Equations in two variables, Progressions, Co-ordinate Geometry)

19. Solve   and $\frac{15}{x+y} - \frac{5}{x-y} = -2$.

$\sqrt{2}+\sqrt{3}$ $\frac{10}{x+y} + \frac{2}{x-y} = 4$ 3x + 7 2x + 3y

20. For which positive value of P does the equations px + 3y – (p – 3) = 0 and 12x + py – p = 0 have infinite solutions.

21. nth terms of Geometric progressions 162, 54, 18,..... and $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}$, ... are equal. Find the value of n.

22. Find area of the triangle (0, 0), (4, 0) and (4, 3) as vertices by Heron's formula.

SECTION - IV (1 x 5 = 5)

(Polynomials, Pair of Linear Equations in two variables)

23. Draw the graph of $y = x^2 + 5x + 6$, hence find its zeroes. Also verify them.

24. Solve by graph $4x - y = 16$ and   .

PART - B

Time : 30 min **Marks : 15**

Note : 1. Each question carries 1/2 mark.

2. Answers are to be written in the question paper only.

3. All questions are to be answered.

4. Marks willnot be given for over written, re-written or erased answers.

I. Write the Capital Letter of the correct answer in the brackets provided against each question.

1. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n x5$ and L.C.M. of a, b, c is $2^3 \times 3^2 \times 5$ then n = ... ( )

   A) 1 B) 2 C) 3 D) 4

2. In the following a polynomial is .... ( )

   A) B) C) D) $x^{-7} + x^2 + x + 8$

3. If the line 3x + 2ky = 2 and 2x + 5y + 1 = 0 are parallel then value of k is .... ( )

   A) B) C) D)

4. If z is added to every terms of an A.P. with common difference 3. Then the common difference of new A.P. is .... ( )

   A) 5 B) 6 C) 3 D) 2

5. If [illegible] then [illegible] ( )

   A) 0 B) 2 C) 4 D) 1

6. The intersection point of x, y - axis is .... ( )

   A) (2, 0) B) (0, 0) C) (0, 2) D) (3, 4)

7. In the following the point equidistant from (2, 0) on x-axis is .... ( )

   A) (–3, 0) B) (7, 0) C) A and B D) (2, 5)

8. The slopes of segments AB and BC are equal then Area of ΔABC = ... ( )

   A) positive B) zero C) negative D) complex

9. In two points, if x co-ordinates are '0' then the slope of line joining the points is .... ( )

   A) 0 B) 1 C) –1 D) undefined

10. In the following which set of points represent a triangle .... ( )

    A) (1,2) (1,3) (1,4) B) (5,1) (6,1) (7,1) C) (0,0) (–1,0) (2,0) D) (1,2) (2,3) (3,4)

Fill up the blanks :

11. In $x = p/q$, prime factor of q is $2^n.5^m$ then x is a .... decimal.

12. If A, B are disjoint sets then $A \cap B =$.........

13. The graph of $y = ax^2 + bx + c$ is called ....

14. Inconsistent pair of linear equations have .... solutions.

15. ....

16. The sum of first n odd numbers is ....

17. The distance of (x, y) from origin is ....

18. The centre is a circle is (–1, 3) and one end point of a diameter is (2, –1), the other end point is ....

19. Father of co-ordinate Geometry is ....

20. Intersection point of the lines x = 0 and y = 0 is ....

III. Match the following. **$10 \times {}^{1}/_{2} = 5$**

i. Group - A **Group - B**

21.	value of	[ ]	A)
22.	logarithm form of	[ ]	B)
23.	2 log 3 = ...	[ ]	C) non terminating recurring decimal
24.	If $\log_4 8 = x$ then x = ...	[ ]	D)
25.	is a	[ ]	E) terminating decimal
			[illegible] F) log 8
			G) $\log_{49} 7 =$
			H) log 9

ii. Group - A **Group - B**

26.	No. of zeroes in the figure	[ ]	A) 0
27.	then p(–3)	[ ]	B) (0, 0)
28.	$a^2y^2 + 4axy + 4x^2$	[ ]	C) 2
29.	origin	[ ]	D) 3
30.	No. of zeros of $x^4 - 16$	[ ]	E) $(ay + 2x)^2$
			F) 4
			G) undefined
